AN INVESTIGATION OF AN EXPANDING ELLIPTICAL EARTH ESCAPE TRAJECTORY FOR A LOW THRUST SPACE VEHICLE

SHELDON DREWS,
ROBERT REILLY GOLDNER
and
ARTHUR EDGAR TREIBER

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by

Sheldon Drews, LT., U. S. Navy
S.B., United States Naval Academy, 1952
S.B., United States Naval Postgraduate School, 1960

and

Robert Reilly Goldner, LT., U. S. Navy
S.B., United States Naval Academy, 1953
S.B., United States Naval Postgraduate School, 1960

and

Arthur Edgar Treiber, LT., U. S. Navy
S.B., University of South Carolina, 1952
S.B., United States Naval Postgraduate School, 1960

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Signatures of Authors ______________________________

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Department of Aeronautics and Astronautics, May 19, 1961

Certified by ______________________________
Thesis Supervisor

Accepted by ______________________________
Chairman, Departmental Committee on Graduate Students

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S. Drews, LT, USN; R. R. Goldner, LT, USN;
and A. E. Treiber, LT, USN

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ABSTRACT

This thesis studies the advantages and disadvantages of an expanding elliptical acceleration maneuver for a space vehicle, in which a low level acceleration is applied tangentially for small arcs of true anomaly centered at perigee. The main advantage of this type of orbit is fuel economy. The elliptical orbit requires less than half the total thrust of an expanding circular spiral using the same acceleration.

The elliptical maneuver requires much greater total time, due to the large amount of non-thrusting time in each orbit. Typical of low thrust acceleration orbits, much of the accelerating time is spent in the high density Van Allen radiation regions. For these two reasons, a vehicle which remains unmanned until escape energy is neared, is considered.

The problem of orbital rendezvous and resupply is considered and deemed feasible using a single stage reuseable ferry vehicle. The acceleration orbit may be stabilized in a polar plane so that, as escape energy is neared, the orbit is completely clear of the high radiation Van Allen regions.

Thesis Supervisor: Paul E. Sandorff

Title: Professor of Aeronautics and Astronautics

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TABLE OF CONTENTS

SYMBOLS

Alphabetical

A	Magnitude of total acceleration
a	Semi-major axis of conic section
$\mathcal{E}$	Energy integral = $\frac{V^2}{2} - \frac{Gm_\oplus}{r}$
e	Eccentricity of a conic section
$Gm_\oplus$	Gravitational constant of Earth = 1.407528×10^{16} (feet/second)^2
g	($=g_o$), gravitational conversion constant, = 32.1740 lbm/slug
I_{sp}	(or I), Specific impulse
i	Angle of inclination of orbital plane to reference plane
M_i	Mass at ignition (initial mass)
M_L	Mass of payload
M_o	Mass at ignition (initial mass)
m	Mass of body
P	Period of orbit
p	Semi-latus rectum of conic section = $a(1-e^2)$
r	Radius of conic from focus to given point
r_α	Radius of ellipse at apogee
r_π	Radius of ellipse at perigee
$r\ddot{b}$	Orthogonal component of acceleration
$\ddot{r}$	Radial component of acceleration
$r\ddot{\theta}$	Circumferential component of acceleration
T	Total thrusting time

SYMBOLS (Continued)

Alphabetical

v	Velocity
v_i	Ideal velocity, a scalar measure of propulsive capability = $g_o I_{sp} \ln (M_i/\text{Mass at burnout})$

Greek

α	Apogee or aphelion. Point in orbit most distant from primary
δ	Dead weight ratio = ratio of sum of structure, engine, and residual masses to Mass at ignition
θ	True anomaly: angular position in orbit measured from perigee
λ	Payload ratio = ratio of mass of payload to mass at ignition
π	Perigee or perihelion, point in orbit closest to primary
ω	Argument of perigee, angular measure eastward from line of nodes

Special

♈	First point of Aries, prime celestial reference direction (the direction of the sun at instant of vernal equinox)
☊	Longitude of ascending node, measured east of Aries.
$\dot{(\ )}$	Derivative with respect to time of a quantity = d()/dt
$\grave{(\ )}$	The additional time rate of change of a quantity due to perturbational effects, called "( ) grave"
$\Delta(\ )$	Change of a quantity

OBJECT

The object of this thesis is to investigate the feasibility, advantages, and disadvantages of an elliptical acceleration orbit for a low thrust interplanetary space vehicle. The desired orbit should incorporate the advantages of a propulsion system of high specific impulse, capable of sustained low thrust for a long duration, yet overcome the disadvantage of the reduced energy addition efficiency of a conventional expanding spiral acceleration maneuver.

CHAPTER 1

INTRODUCTION

1.1 Previous Studies of Low Thrust Acceleration

Many studies of interplanetary travel have been undertaken. A large portion of these studies have dealt with the use of a low thrust propulsion system of high specific impulse to subject the space vehicle to a long duration acceleration of small magnitude. To date most of these studies have dealt with the assembly of the space vehicle in a low earth orbit and the initiation of a continuous low thrust program, once assembly has been completed and the crew placed aboard. Such thrust programs have resulted in ever expanding circular or elliptical orbits. Finally, as energy is added, the escape condition is reached and the space vehicle proceeds on its journey, utilizing the low thrust propulsion system to achieve the desired interplanetary trajectory.

Irving[1] considers several low thrust acceleration trajectories for thrust levels from .0001g to .005g. Although the trajectories vary from circular to elliptical, all are considered for constant thrust (tangential for

circular orbits and non-tangential for elliptical orbits). Perkins[2] also investigates trajectories starting from circular orbits and utilizing tangentially applied thrust to attain escape. Dobrowski[3] investigated escape using radial thrust. Some very important work was done by Tsien[4] and Michelson[5] on optimum trajectories and optimum thrust programs. The fact remains, that all these treatments considered constant thrusting. Camac[6] performed an interesting investigation based on thrusting in the vicinity of perigee with maximum available thrust and utilizing a lower thrust level over certain segments of the elliptical orbit as power was available.

1.2 Purpose of this Investigation

It is the purpose of this thesis to investigate still another type of low thrust acceleration trajectory for a space vehicle. The escape problem begins in a circular orbit at a height of 100 nautical miles. The space vehicle is unmanned, the thrusting program being automatically controlled. It will remain unmanned until escape energy has nearly been achieved. A feature desired of the trajectory is to keep the perigeal altitude nearly coincident with that of the initial orbit in order to maintain the efficiency of energy addition at a high level. The investigation consists of thrusting programs in which the thrusting is done in the vicinity of perigee. It considers the effect of this tangential thrusting on the

orbital parameters, the efficiency of energy addition, the total acceleration time, and the interplay between these parameters.

To this end, the following pages contain studies of various thrusting programs which will achieve the desired orbit, and some of the advantages and disadvantages of the proposed orbit as compared with an expanding circular orbit. Certain aspects of this thesis are treated rather qualitatively, since a full treatment of these aspects would be suitable topics for individual thesis investigations.

In this thesis, the various parameters used are defined in the List of Symbols. The reference frame is the plane of the equator through the Earth, with the primary reference direction as the first point of Aries. The reference plane and the main orbital parameters are shown in Fig. 1-1, for clarity.

Identification of Orbit of a Space Vehicle

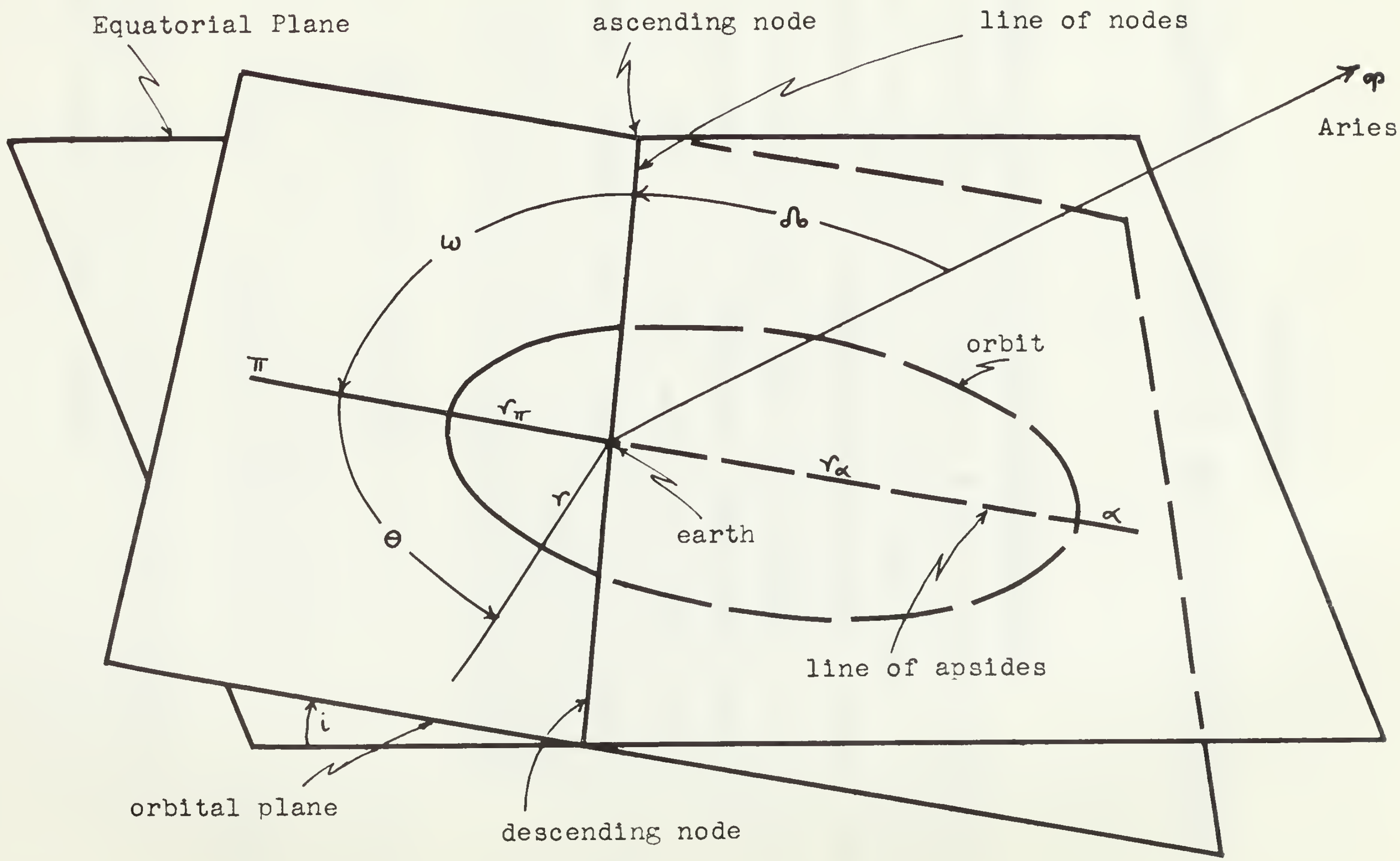


Fig. 1-1

CHAPTER 2

BASIC CONCEPTS OF THRUSTING IN THE VICINITY OF PERIGEE

2.1 Treatment of Low Thrust as a Perturbation

In determining the effects of a low thrust propulsion system on the parameters of an orbit, this thesis considers the magnitude of the acceleration to be small enough to be treated as a perturbation. This investigation is limited to the consideration of tangential thrust and deals with a problem in which all forces and motion are confined to the original orbital plane. Sandorff[7], in a chapter dealing with the general perturbation of an elliptical orbit, presents the following basic perturbation equations:

$$a' = \frac{2a^2}{Gm_\oplus}\left(\dot{r}\dot{r}' + [r\dot{\theta}]\,r\dot{\theta}'\right) \tag{2.1}$$

$$e' = \frac{p}{e\,Gm_\oplus}\dot{r}\dot{r}' + \frac{1}{e\,Gm_\oplus}\left(p - \frac{r^2}{a}\right)(r\dot{\theta})\,r\dot{\theta}' \tag{2.2}$$

From orbital mechanics, the following equations are applicable to an elliptical orbit:

$$r_\pi = a\,(1-e) \tag{2.3}$$

$$p = r_\pi (1+e) \tag{2.4}$$

$$r^2 = \frac{a^2(1-e^2)^2}{(1+e\cos\theta)^2} \tag{2.5}$$

$$(1+e\cos\theta) = (r\dot{\theta})\sqrt{\frac{p}{Gm_\odot}} \tag{2.6}$$

$$\dot{r} = \sqrt{\frac{Gm_\odot}{p}}\; e \sin\theta \tag{2.7}$$

Since all thrust is tangential,

$$\dot{r}' = A\frac{e\sin\theta}{\sqrt{1+e^2+2e\cos\theta}} \tag{2.8}$$

$$r\dot{\theta}' = A\frac{1+e\cos\theta}{\sqrt{1+e^2+2e\cos\theta}} \tag{2.9}$$

Substituting (2.3) through (2.9) into (2.1) and (2.2) and simplifying yields the following perturbative effect of thrust on the semi-major axis and the eccentricity of the orbit:

$$a' = A\frac{2\, r_\pi^{3/2}}{(1-e)^2}\sqrt{\frac{1+e^2+2e\cos\theta}{Gm_\odot\,(1+e)}} \tag{2.10}$$

$$e' = A\;(2)(e+\cos\theta)\sqrt{\frac{r_\pi(1+e)}{Gm_\odot(1+e^2+2e\cos\theta)}} \tag{2.11}$$

Differentiating (2.3)

$$r_\pi' = (1-e)\,a' - e\,a' \tag{2.12}$$

Substitution of (2.3) through (2.9) into (2.12) and rearrangement yields the perturbative effect of thrust on the perigeal radius,

$$r_\pi' = A \frac{2 r_\pi^{3/2}}{\sqrt{Gm_\oplus}} \left[\frac{1 - \cos\theta}{\sqrt{(1+e)(1+e^2+2e\cos\theta)}} \right] \tag{2.13}$$

Sandorff[7] also provides the following equation for the rotation of the line of apsides:

$$\omega' = -\frac{1}{e}\sqrt{\frac{p}{Gm_\oplus}}\,(\cos\theta)\,\mathring{r}' + \frac{(p+r)}{e\sqrt{pGm_\oplus}}\,(\sin\theta)\,r\mathring{\theta}' \tag{2.14}$$

Again, substitution of (2.3) through (2.9) and rearrangement yields the perturbative effect of tangential thrust on the line of apsides,

$$\omega' = A \frac{2}{e^2}\sqrt{\frac{p}{Gm_\oplus}} \left[\frac{e\sin\theta}{\sqrt{1+e^2+2e\cos\theta}} \right] \tag{2.15}$$

It is seen in (2.10), (2.11), (2.13), and (2.15) that the magnitude of the acceleration, "A", which is dependent upon the weight of the space vehicle and the thrust of the propulsion system, remains a factor outside the integral when these expressions are evaluated. In an effort to get meaningful answers and quantitatively compare this analysis with other analyses[1,6] using different thrusting techniques, a value of .001g was selected for the acceleration. The results presented in this thesis are numerically based upon this value of acceleration, but are equally

useable with any magnitude of tangential thrust, by appropriate scaling.

2.2 Integration of the Perturbation Equations

Equations (2.10),(2.11),(2.13), and (2.15) are used as a basis for the calculations of this thesis. Although the perturbation derivatives are taken with respect to time, it would seem more realistic to integrate the equations with respect to the true anomaly. These equations are difficult to evaluate in closed form, but are aptly suited for digital computation. Equation (2.13) is redundant to the solution, but it serves two purposes. First, it directly produces a desired quantity, the change of perigeal altitude due to thrust. Secondly, it serves to determine the degree of error in the final computer results. Use of this redundant equation establishes the probability of error in the results as less than one per cent.

Fig. 2-1 illustrates the change of orbital energy as a function of the thrust in the vicinity of perigee. It is seen that the change of energy, for thrusting arcs smaller than - to + 45 degrees from perigee, is essentially constant and independent of the eccentricity. This is explained since the energy integral is defined as:

$$\mathcal{E} = \frac{v^2}{2} - \frac{Gm_\oplus}{r} \tag{2.16}$$

Differentiating (2.16),

$$\mathcal{E}' dt = \left(v v' + \frac{G m_{\oplus} r'}{r^2} \right) dt \tag{2.17}$$

For small changes r, the factor r'/r^2 is much smaller than vv', and may be neglected. Since the velocity increases with increasing eccentricity, while the time of traversing a given thrusting arc decreases with increasing eccentricity, the product vv' dt remains essentially constant and independent of eccentricity.

Fig. 2-2 illustrates one major benefit to be derived from using an elliptical acceleration maneuver. From (2.17), it is seen that the maximum rate of change of orbital energy for a given acceleration is derived by applying the acceleration at the highest possible velocity and at the lowest possible radius. If the term "energy efficiency" is defined as the amount of energy added to the orbit during a one second thrusting increment, then Fig. 2-2 demonstrates the energy efficiency of the elliptical acceleration maneuver. For orbits of nearly constant perigeal altitude, the energy efficiency increases with increasing eccentricity. An expanding circular orbit would begin acceleration at the same energy efficiency as noted by the curve for eccentricity equal to 0.01 in Fig. 2-2. Instead of increasing, however, the energy efficiency would decrease due to the decreasing velocity and the increasing radius of the expanding circular orbit.

2.3 Perturbation Effect of Thrust on Eccentricity of the Orbit

Fig. 2-3 illustrates the change of eccentricity of the orbit as a function of the thrusting arc. The form of these curves is similar to the form of the curves of Fig. 2-1, the main deviation occurring at low values of eccentricity. The similarity between the two figures is obvious if (2.17) is rewritten in the form

$$\mathcal{E}'\,dt = \frac{Gm_{\oplus}}{2r_{\pi}}\left[e' + (1-e)\frac{r_{\pi}'}{r_{\pi}}\right]dt \tag{2.18}$$

It is thus seen that the change in energy and the change in eccentricity are proportional, except for the second modifying term. This term is significant, even for small changes in perigeal radius, at very low values of eccentricity. As eccentricity increases, however, the modifying term diminishes to a negligible value.

2.4 Minimizing the Change of Perigeal Altitude

The main purpose of this analysis is to investigate an acceleration maneuver in which the energy efficiency is maintained at a high level by keeping the increase of perigeal altitude to a minimum. To this end, Fig. 2-4 illustrates that the thrusting arc must be kept fairly small in order to minimize the increase of perigeal altitude.

In order to validate the accuracy of all results

presented in this chapter, a necessary condition is that the change of perigeal radius be limited to ten per cent forthe entire acceleration maneuver. This amounts to a maximum change of perigeal altitude of 350 nautical miles (2,128,000 feet). This criterion places real restrictions on the thrusting arc which may be used, thus restricting the amount of energy which may be gained during any one orbit for a given acceleration. This restriction effectively determines the least number of orbits required to effect the acceleration to escape.

As seen in Fig. 2-4, the change of perigeal altitude over a full - to + 100 degree thrusting arc is independent of the eccentricity of the orbit. That the change of perigeal altitude decreases with increasing eccentricity for small thrusting arcs, yet the perigeal change is constant with eccentricity for the entire - to + 100 degree thrusting arc, is an apparent paradox. It is due to the fact that the thrusting time for a given arc is a hidden parameter. For small thrusting arcs (less than - to + 60 degrees), it is apparent from Fig. 2-5 that the perigeal increase is reduced as eccentricity is increased. This is due to the fact that the radius of curvature of the thrusting arc increases with increasing eccentricity, thereby causing the tangential thrust to be directed more nearly perpendicular to the line of apsides. This reduces the effect of the acceleration on changing the perigeal altitude and increases the effect on changing the perigeal

velocity. This is coupled with the fact that, at high values of eccentricity, the velocity throughout the - to + 60 degree segment is increased, and the thrusting time correspondingly decreased. The increase of the linear distance of the arc subtended for these thrusting arcs is small and may be neglected. From 60 to 90 degrees from perigee, however, the greater increase in the linear distance of the arc subtended increases the thrusting time of the increment, overcoming the effect of the increased velocity. This fact is illustrated in Fig. 2-1 and Fig. 2-4.

2.5 Use of the Results in Planning an Acceleration Maneuver

Utilization of a - to + 100 degree thrusting arc would result in an average energy change of 0.16×10^7 feet^2/sec^2 per orbit, and would require 198 orbits to achieve escape energy. For a perigeal altitude change of 76,000 feet per orbit, the total change of perigee would be 15,000,000 feet, or 68% of the original radius. Since this solution is incompatible with the terms of the problem, and outside the realm of accuracy of the results, the main indication is that very large thrusting arcs must be avoided at low values of eccentricity.

Fig. 2-5 is a plot of the perigeal change for thrusting arcs of - to + 60 degrees. This figure illustrates that the change of perigeal altitude decreases rapidly with increasing eccentricity for a fixed, small thrusting arc. As a result, larger thrusting arcs are permissible at high values of eccentricity.

As an example, consider an acceleration maneuver in which the orbital energy must be increased from -31.5×10^7 $\text{feet}^2/\text{sec}^2$ to -1.5×10^7 $\text{feet}^2/\text{sec}^2$ (the latter corresponding to an eccentricity of 0.95). By iteration, using Fig. 2-1 and 2-5, it is found that by thrusting for an arc of - to + 57 degrees throughout the acceleration maneuver, the maximum allowable change of perigeal radius (2,230,000 feet) is achieved and the entire maneuver would require 200 orbits. Since this solution gives the maximum allowable change of perigee, it would seem prudent to use a much smaller thrusting arc at low eccentricity and a larger thrusting arc at high eccentricity, thereby gaining the additional advantage of high energy efficiency at high eccentricity. Such a program would achieve the acceleration maneuver in approximately the same number of orbits, but would have a smaller increase in the perigee. At the same time, fewer orbits at high eccentricity would decrease the total time of the maneuver. As an illustration of this fact, from Fig. 2-5, at an eccentricity of 0.01, a - to + 15 degree thrusting arc changes the perigee by 750 feet and, from Fig. 2-1, imparts a change of energy of 0.036×10^7 $\text{feet}^2/\text{sec}^2$ to the orbit. Similarly, at an eccentricity of 0.95, a thrusting arc of - to + 30 degrees imparts an energy change of 0.076×10^7 $\text{feet}^2/\text{sec}^2$ for the same 750 foot change in perigee. The foregoing examples serve to illustrate the utility of the accompanying curves. The full utilization of similar curves is more fully covered in

Chapter 3, concerned with methods of planning the entire acceleration maneuver to conform with certain limitations.

2.6 Movement of the Line of Apsides

Three parameters are sufficient to establish the shape and orientation of an elliptical orbit in a plane. The two parameters discussed thus far are the perigee and the eccentricity, which are sufficient to determine the shape of the orbit. To determine the orientation, the position of the line of apsides must be known. For an interplanetary mission where the departure point and direction must be calculated, or for escaping the high density radiation regions of the Van Allen belts, it is probable that some adjustments to the line of apsides will have to be made.

In entering the acceleration maneuver, it is doubtful that a perfectly circular orbit could be established from which the perigee of the final acceleration orbit could be arbitrarily selected. Considering the worst case in which the initial orbit is slightly eccentric with its perigee located 180 degrees from the desired position, two solutions are possible. One would be to apply thrust so as to establish a circular orbit, then apply thrust to establish the perigee at the desired point and to reestablish a small eccentricity.

Fig. 2-6 illustrates that a rotation of the line of apsides may easily be performed during the acceleration maneuver itself, without the corresponding loss of efficiency present in the first solution. By thrusting unsymmetrically,

on one side of perigee or another, at very low values of eccentricity, the line of apsides may be conveniently rotated in either direction. Since other considerations limit the amount of thrusting arc which may be used at low values of eccentricity, a number of orbits would be required to achieve the desired rotation. However, the rotation could be achieved with but a small increase in total time. It must be emphasized that rotation of the line of apsides must be accomplished at low values of eccentricity. Final minute adjustments as the maneuver approaches escape are possible, but only at the expense of a considerable increase in the total time.

2.7 Non-planar Perturbation Considerations

Since this investigation deals mainly with the planar case of an acceleration maneuver, no mention has been made of the remaining parameters needed to establish the orientation of the orbit in three dimensional space, namely the inclination and the line of nodes. Sandorff[7] provides the following perturbation equations for these two parameters as follows:

$$i' = \frac{r}{\sqrt{Gm_{\oplus} p}} \cos(\theta + \omega)\, r\dot{b}' \qquad (2.19)$$

$$\Omega' = \frac{r}{\sqrt{Gm_{\oplus} p}} \frac{\sin(\theta + \omega)}{\sin i}\, r\dot{b}' \qquad (2.20)$$

If orthogonal thrusting is used to rotate the line of nodes, there is also an apparent effect on the angle ω,

measured from the ascending node to perigee. The line of apsides does not move with respect to inertial space, but due to the definition of ω, relative to the line of nodes, the effect of orthogonal thrusting is:

$$\dot{\omega} = -\frac{r}{\sqrt{Gm_{\oplus} p}} \frac{\mathrm{CTN}\,(\theta + \omega)}{\sin i} \, r\dot{b} \qquad (2.21)$$

It would appear that the optimum place for the application of ortogonal thrust, to change either or both of these parameters, would be that section of the orbit within 90 degrees of apogee, where the radius is large and the angular function has a large value. Any ortogonal thrust applied to change these parameters would have no effect on the planar parameters discussed previously, except for the apparent effect on ω. Since there would normally be no thrusting within 90 degrees of apogee, in the plane of the orbit, all of this arc is available for full thrust in the orthogonal direction, should adjustments be required.

Change of Orbital Energy per Orbit for Constant Tangential Thrust of .001g.

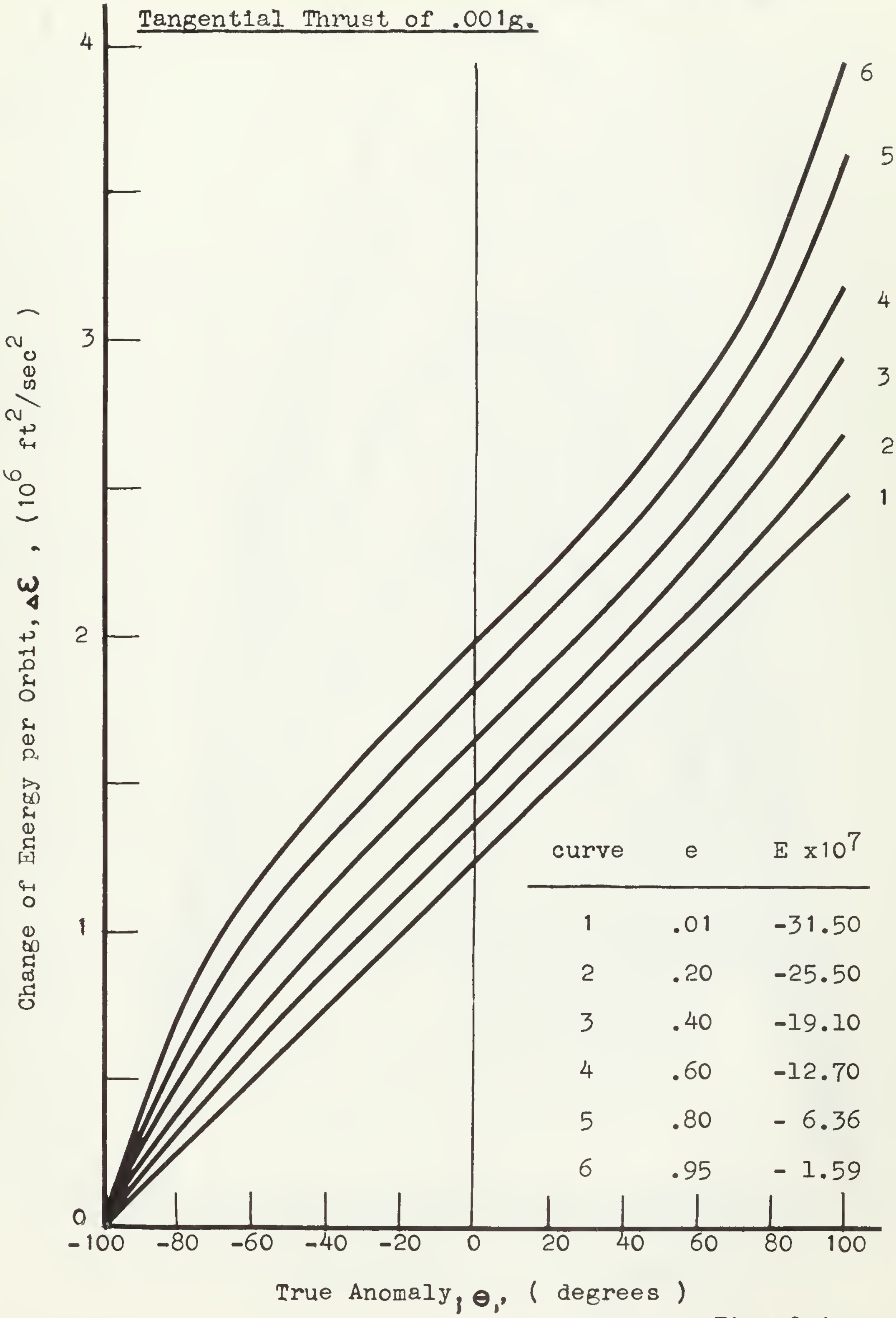


Fig. 2-1

The Change of Energy per Unit Time per Orbit for a Constant Tangential Thrust of .001g

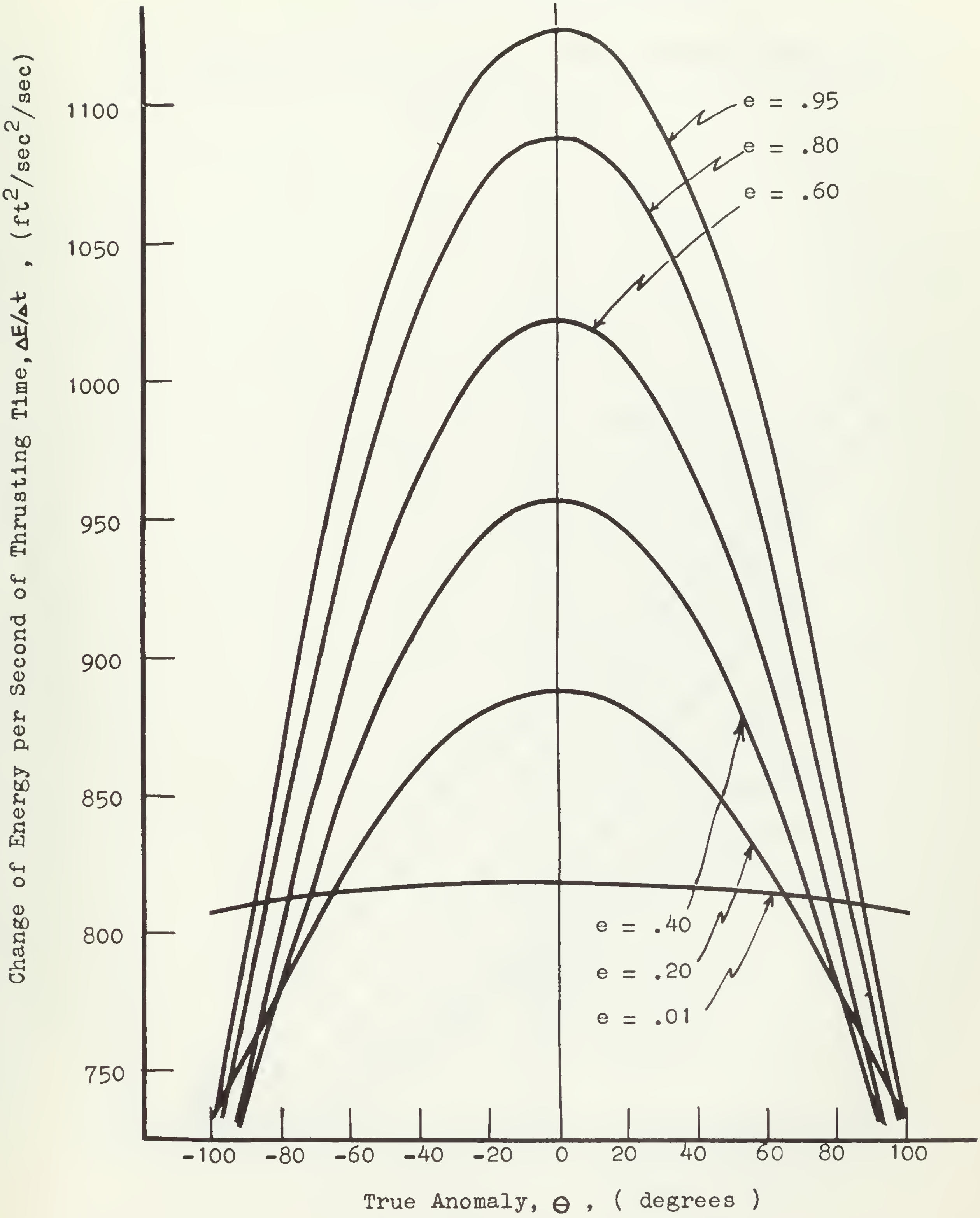


Fig. 2-2

The Change of Eccentricity per Orbit Constant Tangential Thrust of .001g

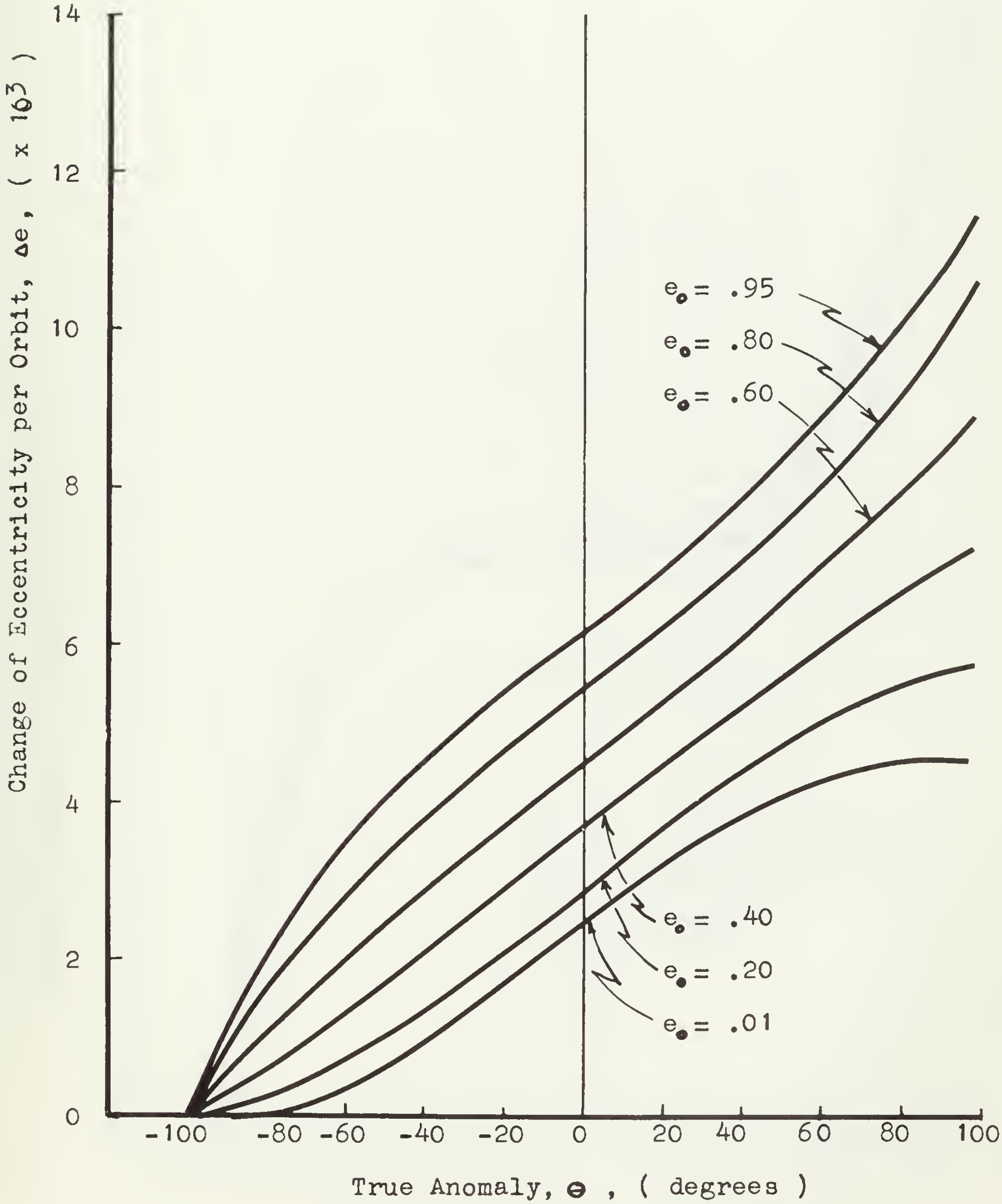


Fig. 2-3

The Change of Perigeal Radius per Orbit for Constant Tangential Thrust of .001g

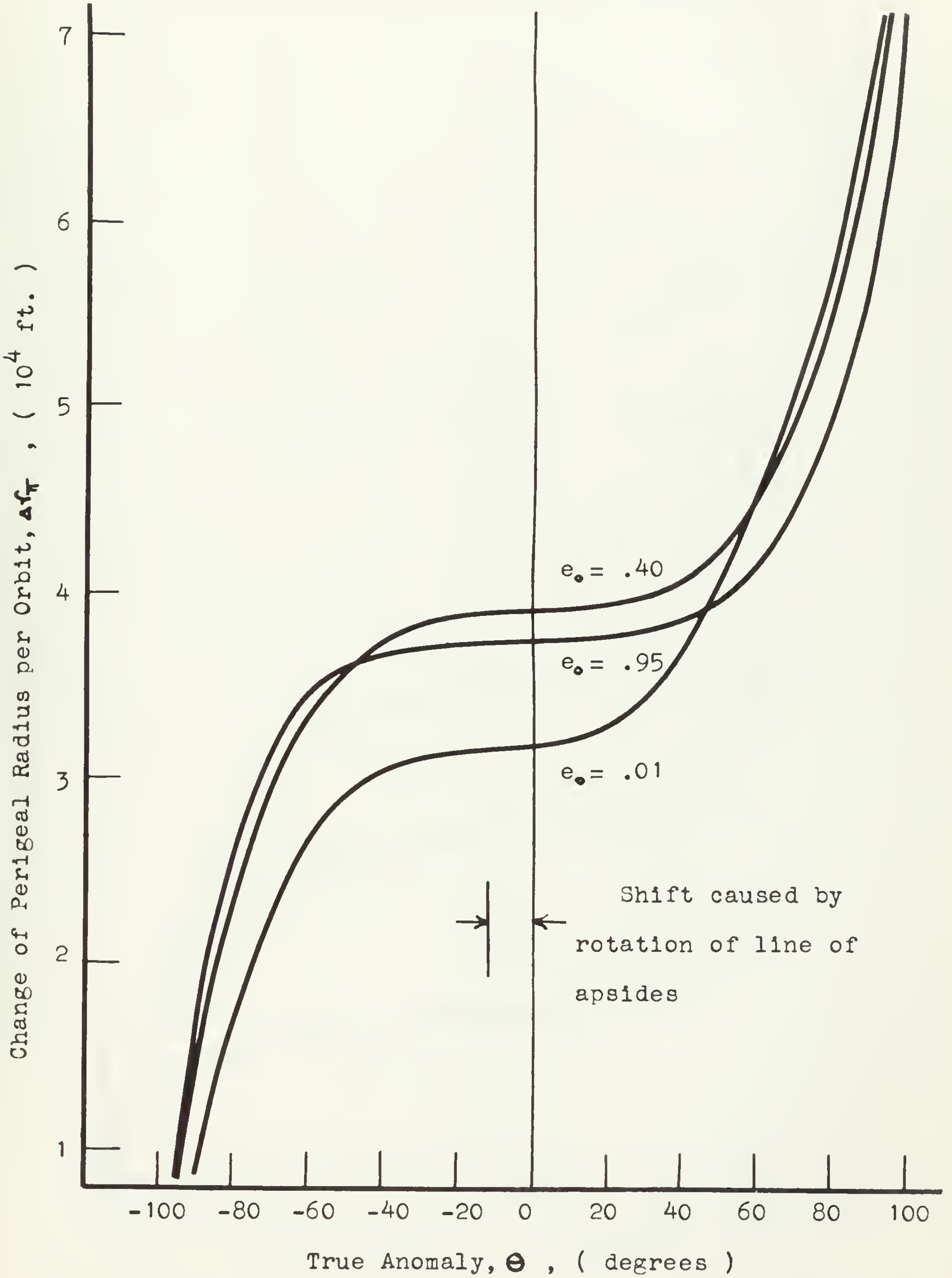


Fig. 2-4

The Change of Perigeal Radius per Orbit for Small Thrusting Arcs for Tangential Thrust of .001g

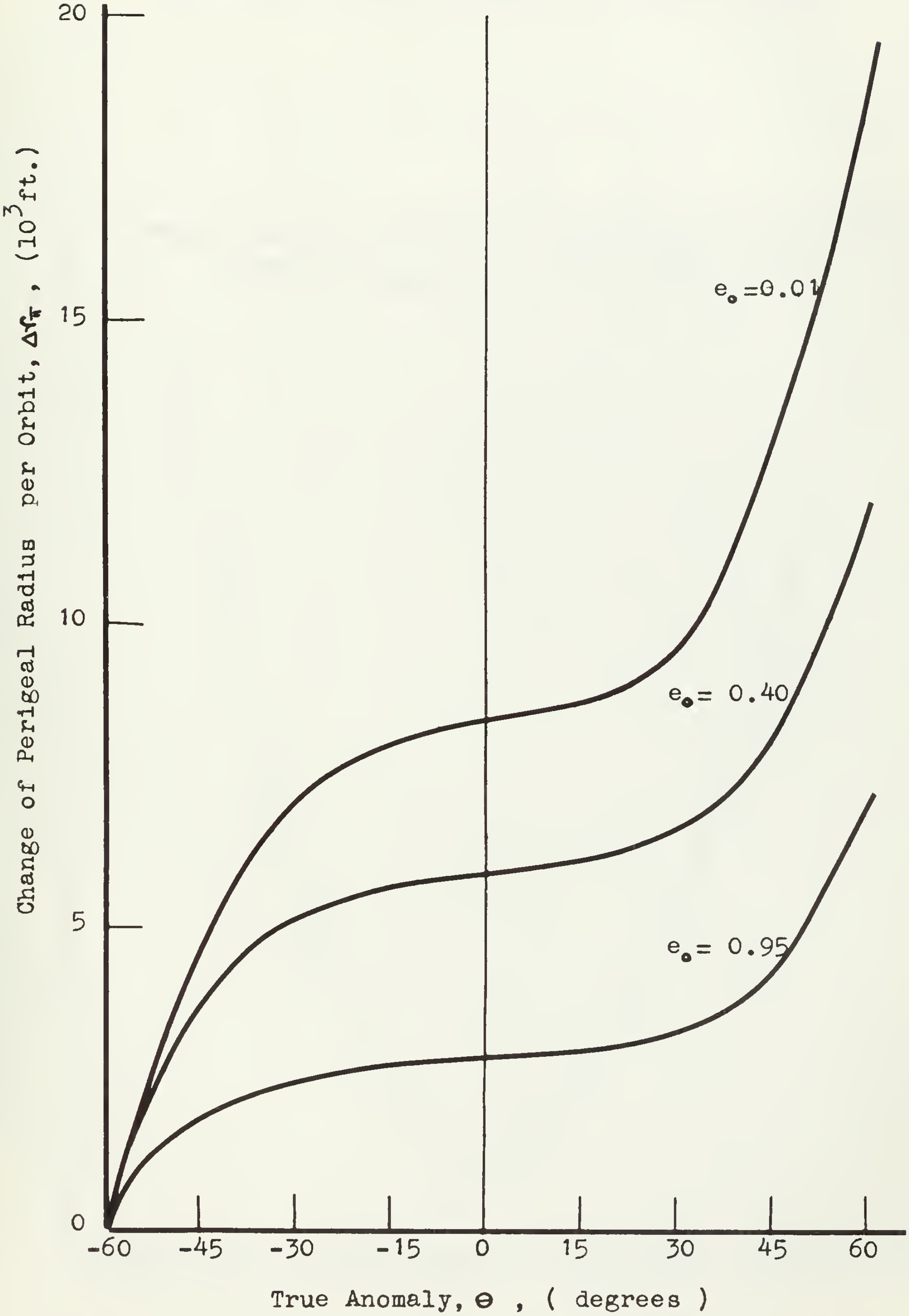


Fig. 2-5

Rotation of Line Of Apsides per Orbit due to Constant Tangantial Thrust of .001g.

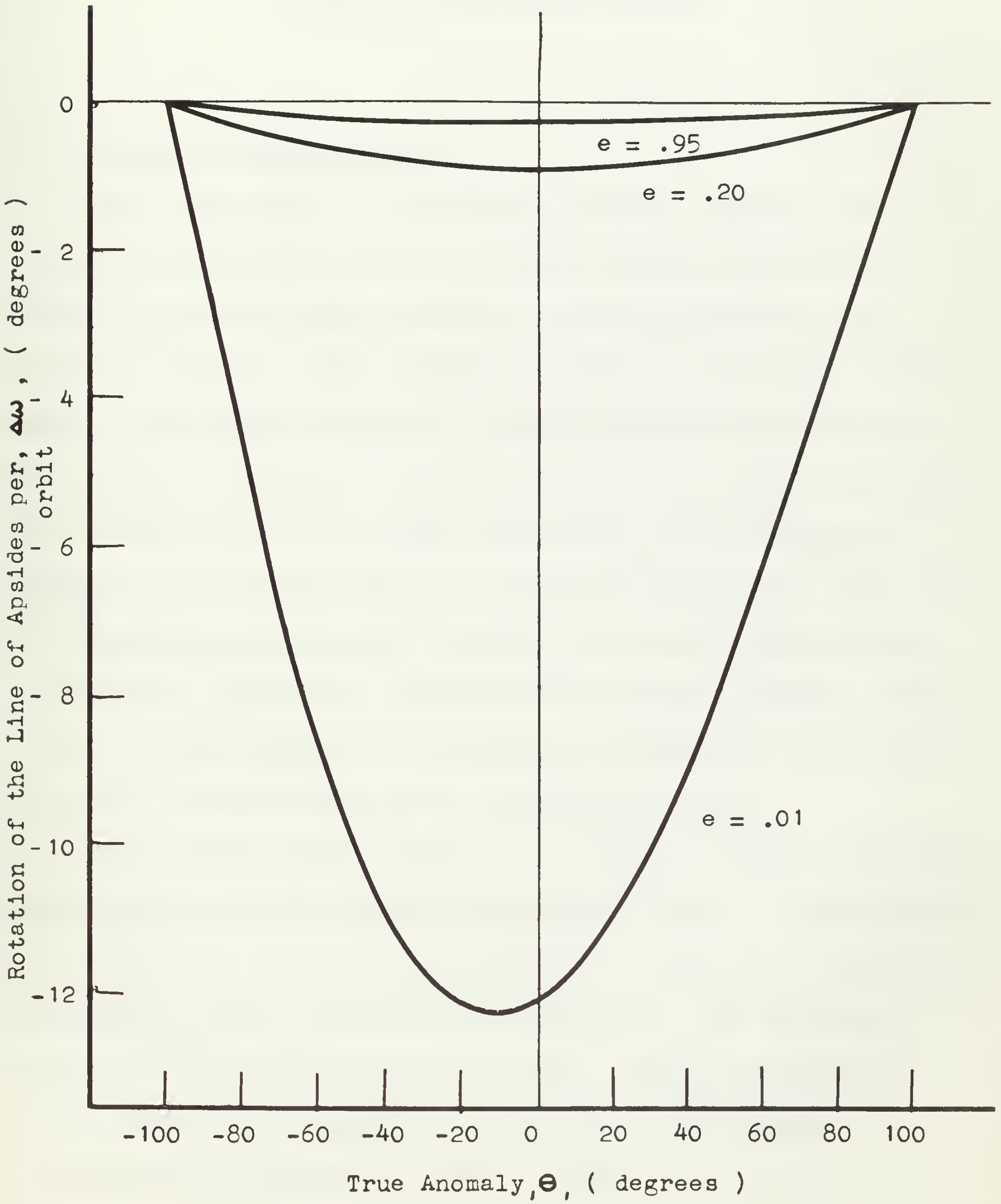


Fig. 2-4

CHAPTER 3

THE ACCELERATION MANEUVER

3.1 General Considerations

The procedure of thrusting through a limited arc centered at perigee of an elliptical orbit, instead of thrusting continuously around the orbit, produces two desired effects. For a given increment of orbital energy added, the thrust required is decreased due to the high thrusting efficiency, and the increase in perigeal altitude is lessened due to the fact that most of the thrusting is being done perpendicular to the line of apsides. This type of thrusting program also produces an undesirable effect, in that the total time required to raise the orbital energy level a given amount is increased. The smaller the thrusting arc, the more prevalent are these effects.

To illustrate the comparison between the two different acceleration maneuvers being compared, Fig. 3-1 and Fig. 3-2 are included. Fig. 3-1 is a diagram of the more conventional expanding spiral acceleration maneuver[1]. The starting orbit is a low circular Earth orbit. Thrust is applied continuously in a tangential direction. The effect is a continuously expanding circular spiral.

Fig. 3-2 illustrates the elliptical acceleration maneuver. The starting orbit is also a circular low Earth orbit. In this maneuver, tangential thrust is applied in a small arc centered about perigee, and the result is an ever expanding ellipse, the perigee of which remains essentially constant throughout the maneuver. Because of the nature of the low thrust acceleration, it is necessary to impose a small increment of velocity by impulsive thrust in order to establish the perigee at the correct position, and to give the orbit a very small value of eccentricity, before the tangential low thrust acceleration can begin.

Since the main consideration here is the use of a small thrusting arc, centered about perigee, the thrusting arcs will be referred to as an arc of "- to + 15 degrees", and should be taken to mean a 30 degree thrusting arc extending from -15 to +15 degrees true anomaly. In the Figures, a curve labeled - to + 15 degrees, in fact, means the same 30 degree arc, symmetrical about perigee.

The change of parameters of the orbits obtained by thrusting over arcs - to + 15 to - to + 90 degrees have been plotted for this chapter with eccentricity as the independent variable, to show how these changes influence the orbit as the orbit expands. The increase of energy per orbit, for various thrusting arcs, is plotted in Fig. 3-3. At high values of eccentricity, thrust is being applied over a longer linear arc and at a higher velocity than at low eccentricities, resulting in a greater increase in

energy for a given thrusting arc.

Fig. 3-4, the plot of the thrusting time per orbit for various thrusting arcs, shows that for a thrusting arc of - to + 30 degrees, the time of thrusting per orbit decreases with eccentricity. This is due to the fact that at high eccentricity, although the length of the arc has increased, the velocity at which the arc is traversed has increased at a greater rate. As the thrusting arc is increased, there is a greater relative increase in the length of the arc with increasing eccentricity, so that, at a thrusting arc of - to + 90 degrees, the thrusting time increases with increased eccentricity.

Referring once again to Fig. 3-3, it is seen that at high eccentricity, a fixed increment of thrust will provide a greater energy increment than it would have provided at a lower eccentricity (and lower velocity). The increment of energy added per orbit when thrusting over a - to + 15 degree thrusting arc remains constant, however, since the effect of the decreased thrusting time offsets the effect of increased velocity as the eccentricity is increased. As the thrusting arc is increased, the decreased thrusting time can no longer balance the increased velocity. Consequently, the energy added per orbit increases with the thrusting arc as well as with the eccentricity.

The increase in the perigeal altitude per orbit, for various thrusting arcs, is plotted in Fig. 3-5. It shows that there is an increasing increase of perigeal altitude

for an increase in thrusting arc and a diminishing increase of perigeal altitude for increasing eccentricity. This is due to the fact that the increase in perigeal altitude is a function of the direction of thrust relative to the line of apsides.

The period of an orbit as a function of eccentricity for constant perigeal altitude is plotted in Fig. 3-6. The period is directly proportional to the semi-major axis to the 3/2 power. The semi-major axis is a function of the perigeal radius and the eccentricity. Since the perigeal altitude is nearly constant, the period is approximately proportional to the eccentricity to the 3/2 power.

3.2 Digital Computer Procedure

In order to demonstrate the expected results of a thrusting program using small thrusting arcs centered at perigee, digital computer programs were run, according to the equations presented in Chapter 2, to simulate actual programs of thrusting. Four programs were run for four different thrusting arcs, with a constant thrusting arc used for each program. The problem was started each time with the initial circular orbit of perigeal radius of 22,117,000 feet, equivalent to a perigeal altitude of 100 nautical miles. A small impulsive thrust at the desired perigee was postulated to give the orbit an eccentricity of 0.0079437 (this was the result of a 100 foot per second velocity increment). Each program was cut off as the orbit reached an eccentricity of 0.92.

At this eccentricity, the non-thrusting time in each orbit amounts to over 98% of the total orbital period. It is considered that a program of this type would be terminated at an eccentricity at or below a value of 0.92, because continued small arc thrusting imposes too great a penalty in total time consumed. Since, at the final perigeal altitude and at an eccentricity of 0.92, the orbit needs only 400 feet per second additional velocity at perigee to escape, two methods may be used to achieve escape. One method would be to use a conventional chemical engine to impulsively add the necessary velocity at perigee. If the low thrust engine is utilized, the alternate method would use a large enough thrusting arc (as close to perigee as possible) to impose the necessary energy to achieve escape. This escape thrust would not have the efficiency demonstrated throughout the program, but it is considered that less efficient thrusting is acceptable in return for the saving in total time.

At an eccentricity of 0.92, the orbit is very close to having escape energy, and this is the logical place to terminate the acceleration maneuver. After the orbit has reached this energy level, the crew would be placed aboard, and orbital resupply and refuelling would take place. After the completion of crew placement and orbital resupply, the decision to depart on the flight would be made, and the escape maneuver performed.

3.3 Discussion of Computer Results

Several conclusions may be made from a study of the results presented in Table 3-1. To optimize the program, for total time, the evident solution is the application of continuous thrust. The result is the expanding spiral maneuver. The other parameter which may be considered is the efficiency of energy addition. As the thrusting arc is decreased, the same energy is added by successively smaller thrusting times (less energy expended by the vehicle propulsion system). The optimum for this criterion is, of course, the conventional chemical method of impulsive thrust. The main conclusion is, therefore, that this program is not an optimum in itself, but represents a desirable compromise between the two optimum programs mentioned.

The main trade-off present in this program is the trade of total time in return for efficiency. As seen by a comparison between the - to + 60 degree program and the continuous thrust program, a saving of 54.5% in the propulsive time (and a corresponding saving in the expelled mass of fuel) results in a program taking 7.55 times as long in total time. This comparison is made, however, under the assumption that the energy input to the engine will provide the same thrust (and acceleration) under small arc thrusting as with continuous thrusting. Camac[6] proposed that more thrust could be achieved by the use of energy storage; that is, by the production of electrical power utilizing solar cells and storage of this power in batteries

until it is needed. Using this procedure, some of the power limitations of the low thrust propulsion system might be overcome, resulting in a greater efficiency of the engine. According to Camac[6], this could result in a much higher thrust level for short periods of time (as in small arc thrusting) than could be sustained continuously. If this theory is true, the total time of the maneuver would be correspondingly reduced, further enhancing its desirable features. At the present state of the art, no definite statement may be made, but will have to be deferred until such time as actual test may be made.

3.4 Use of Graphs to Estimate the Result of a Complete Small Arc Thrusting Program

Although the results presented in Table 3-1 are for thrusting arcs which do not vary throughout the entire program, this type of acceleration maneuver is by no means restricted to the use of the same thrusting arc throughout the program. A relatively simple method for deriving the results of any small arc thrust program is presented here, utilizing the Figures included with this chapter. Fig. 3-7 is a plot of the change of eccentricity per orbit as a function of eccentricity and is included for use in the following method:

1. Divide the acceleration maneuver into segments of either changes of eccentricity, changes of orbital energy, or changes of perigeal radius. The parameter selected is henceforth referred to as "the parameter".

2. Using either Fig. 3-3, 3-7, or 3-5 as applicable, find the average change per orbit of the parameter, for the chosen thrusting arc over the selected segment.

3. Divide the desired change of the parameter by the change per orbit found in step 2. (If the perigeal altitude is selected as the parameter, an iterative procedure will have to be used.) The result is the required number of orbits to effect the desired change of the parameter.

4. From Fig. 3-7, find the average change of eccentricity per orbit and multiply by the number of orbits required to find the total change of eccentricity over the segment. (omit this step if eccentricity is the parameter)

5. Using the segment of eccentricity from step 4, obtain the average thrusting time per orbit from Fig. 3-4, the average change in perigeal altitude per orbit from Fig. 3-5, and the average period per orbit from Fig. 3-6.

6. Multiply the changes of step 5 by the total number of orbits required in the segment to get the resulting changes of various quantities over the segment.

7. Add up the results of all segments to obtain the change in perigeal altitude and the resulting total time and thrusting time for the entire maneuver.

The accuracy of this method was verified by executing the above steps and comparing the results with the computer results of Table 3-1. Using six segments of eccentricity as the parameter, the results were found to agree with the accurate computer results of Table 3-1 to within 3% in the change of perigeal altitude and thrusting time required, and within 10% in the estimate of total time. A greater degree of accuracy may be obtained by dividing the overall program into a greater number of smaller segments. Use of this method is not restricted to thrusting arcs of - to + 60 degrees, but the use of greater thrusting arcs throughout the program results in greater than a 10% increase in the perigeal radius, and moves the problem out of the realm of accuracy of the Figures in this thesis. The described method does not restrict a thrusting program to perigeal changes of less than 10%, but the initial assumption that the perigeal change would not exceed 10% does.

3.5 Corrections to the Orbital Period

Since Fig. 3-6, the period as a function of eccentricity, is based on the assumption of a constant perigeal altitude, the degree of its error is proportional to the increase of perigeal altitude allowed in the program. Since the error in the period is caused by the assumption of a lower perigeal altitude than is actually present, the period will appear to be shorter than the actual period. For reasonable changes of perigeal radius, the range and accuracy of Fig. 3-6 can be increased by applying a correction factor based on the change of perigee.

Let the total change of perigee up to the point of calculation divided by the original perigee be δ . It can be shown that expanding the expression for the period, adding in an error in perigee, expanding, and neglecting terms of higher order, that the error in the period is:

$$\epsilon = \frac{3}{2}\delta - \frac{3}{8}\delta^2 + \frac{1}{16}\delta^3 + \cdots \tag{3.1}$$

and the actual period is:

$$P_{actual} = (1 + \epsilon)\, P_{Fig.\ 3\text{-}6} \tag{3.2}$$

Escape from Circular Orbit Using Constant Tangential Thrust

Acceleration of .001g

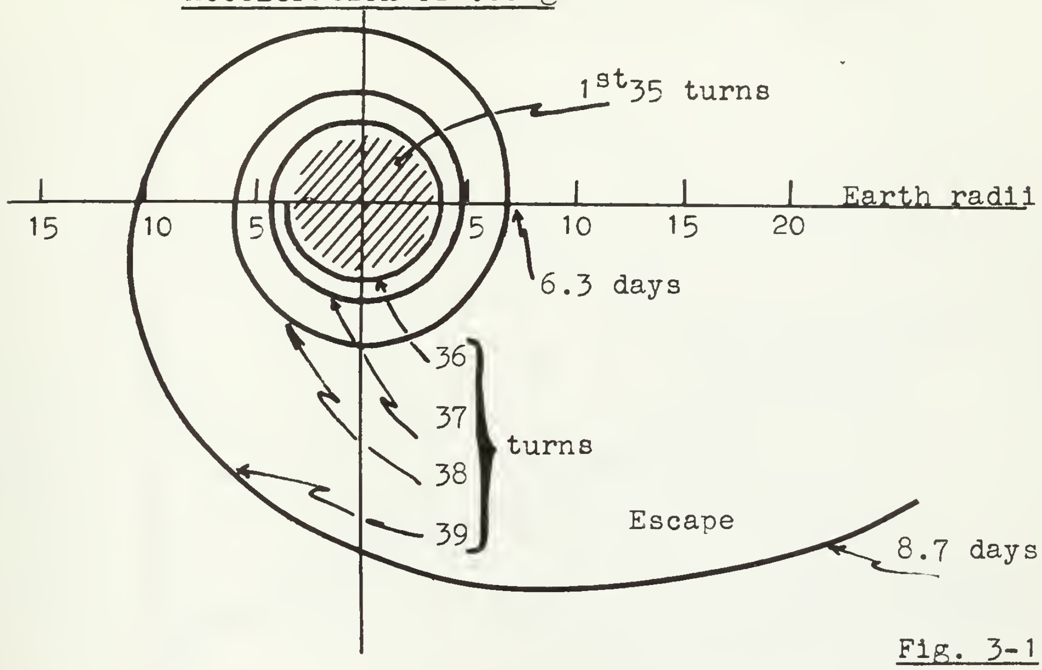
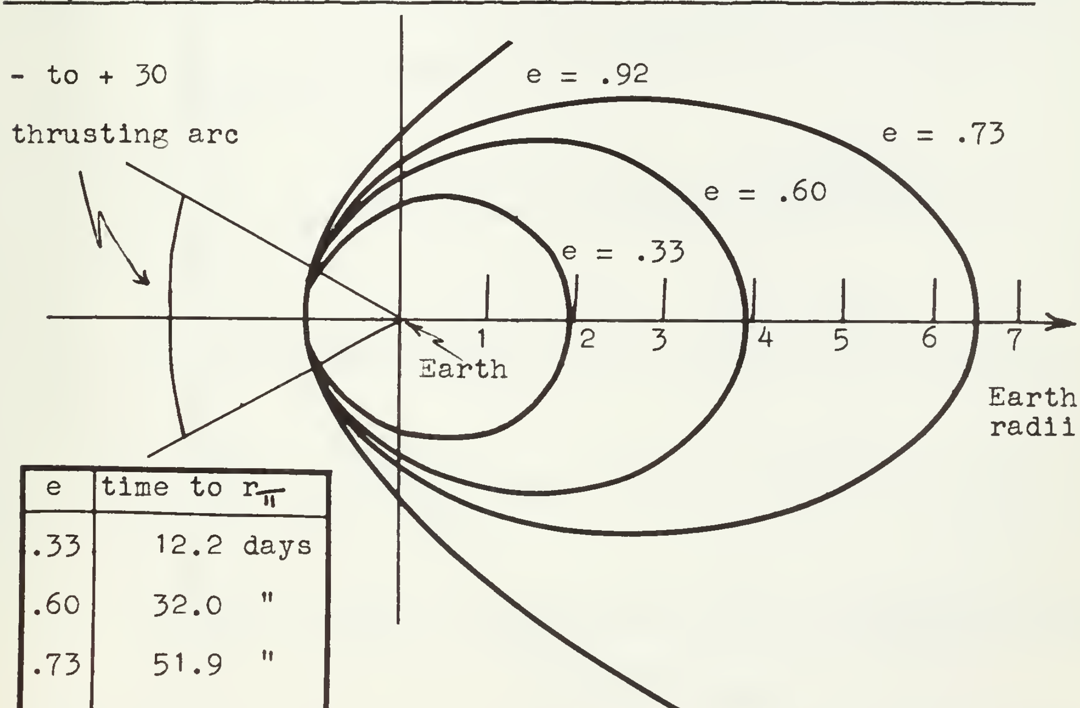


Fig. 3-1

Escape from Circular Orbit Using the Elliptical Escape

Trajectiory with Constant Perigeal Altitude (- to + 30)



e	time to r_π
.33	12.2 days
.60	32.0 "
.73	51.9 "
.92	137.2 "

Fig. 3-2

The Increase in Energy per Orbit as a Function of Eccentricity for Various Thrusting Arcs

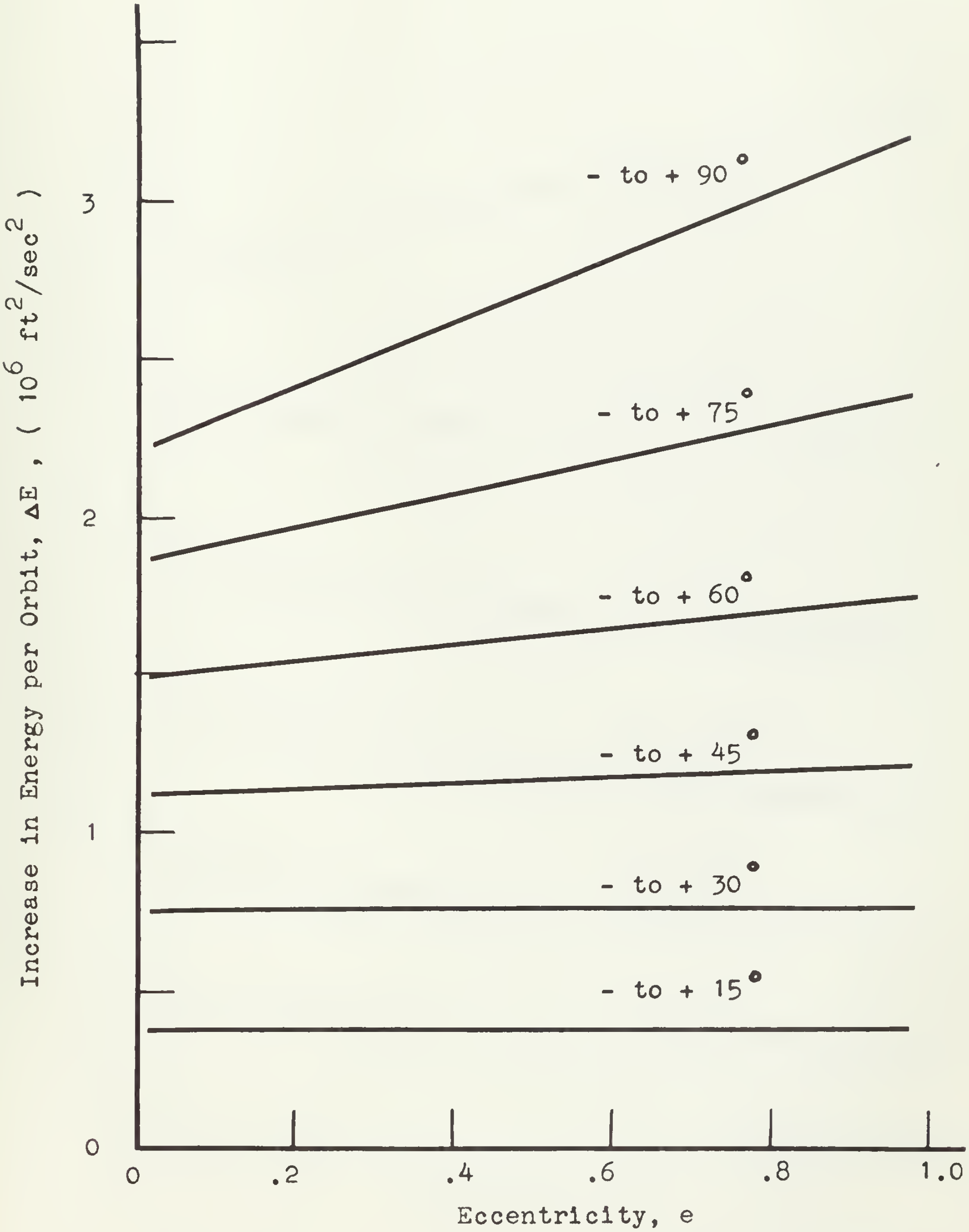


Fig.3-3

The Time of Thrusting per Orbit as a Function of Eccentricity for Various Thrusting Arcs

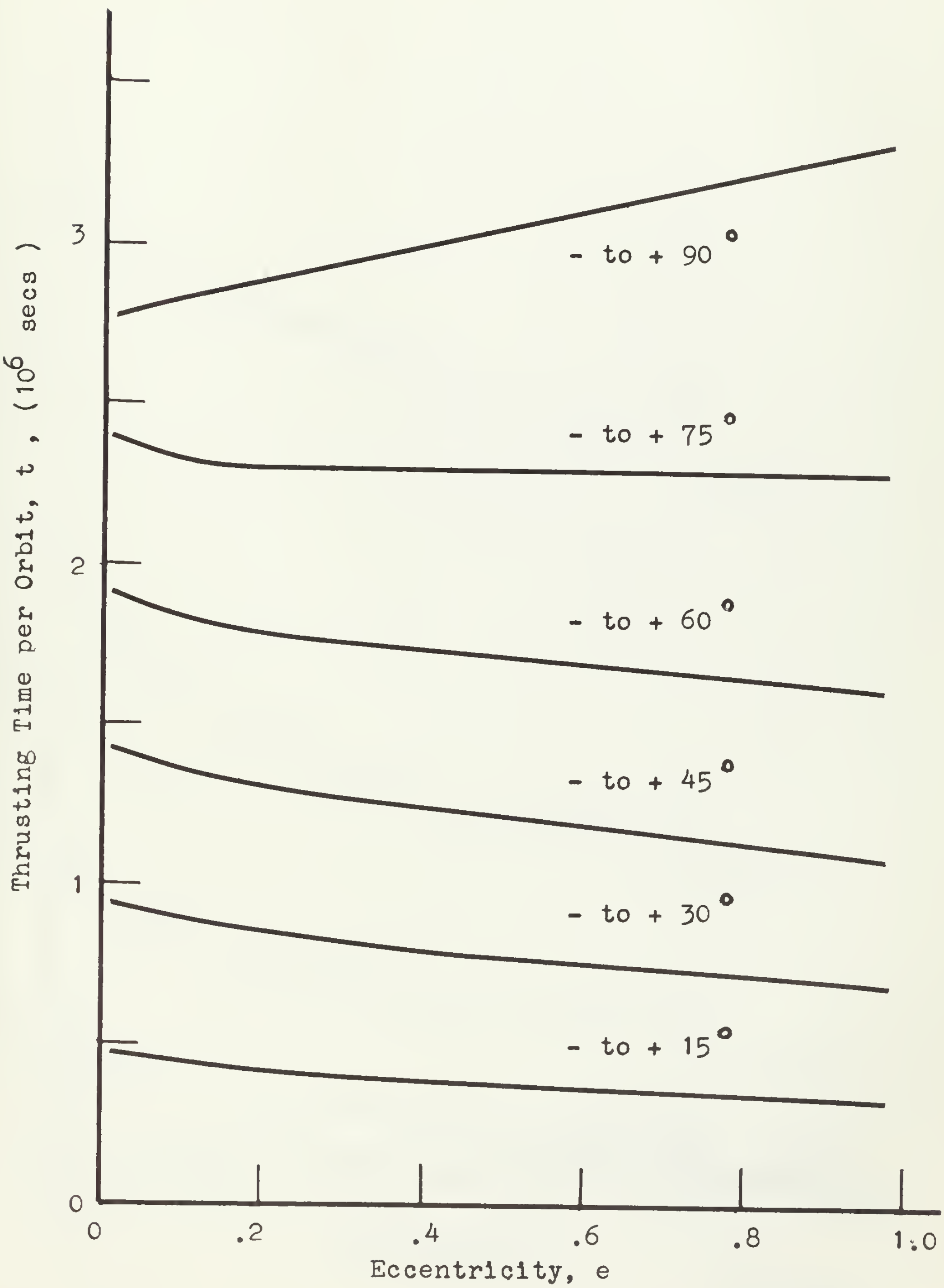


Fig. 3-4

The Increase in the Perigeal Altitude per Orbit as a Function of Eccentricity for Various Thrusting Arcs

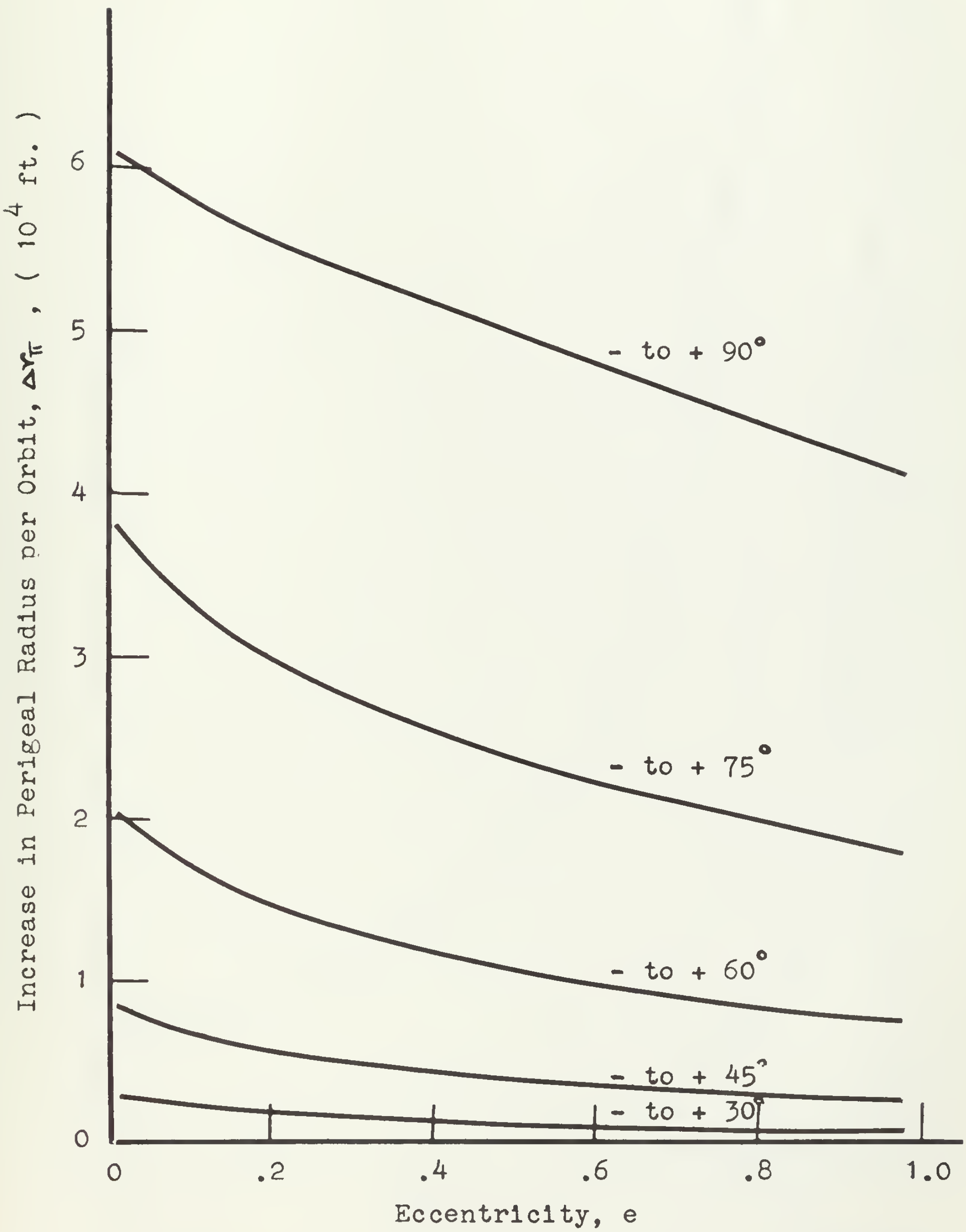


Fig. 3-5

Period of the Ellipse as a Function of Eccentricity

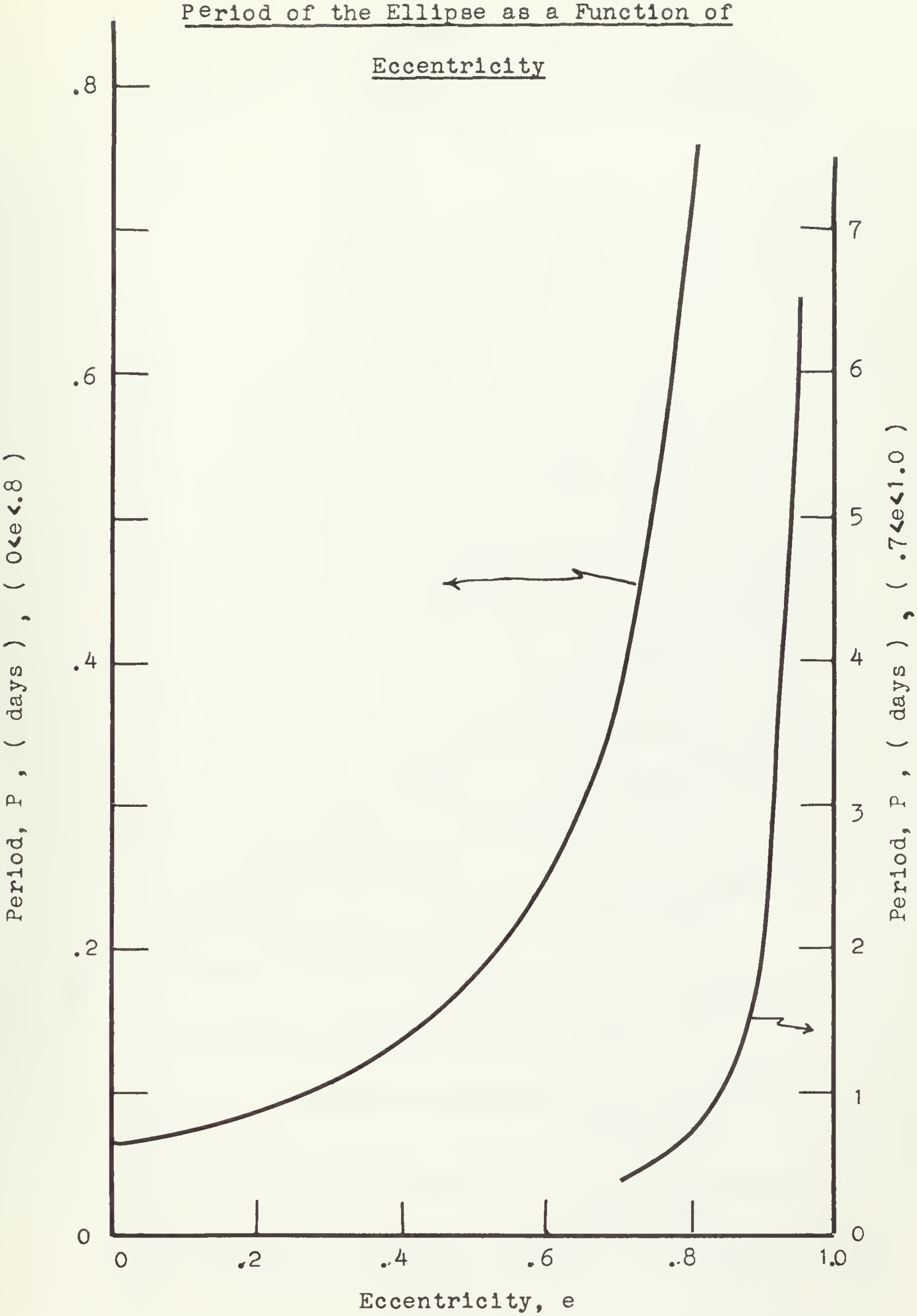


Fig.3-6

The Change in Eccentricity per Orbit as a Function of Eccentricity for Various Thrusting Arcs

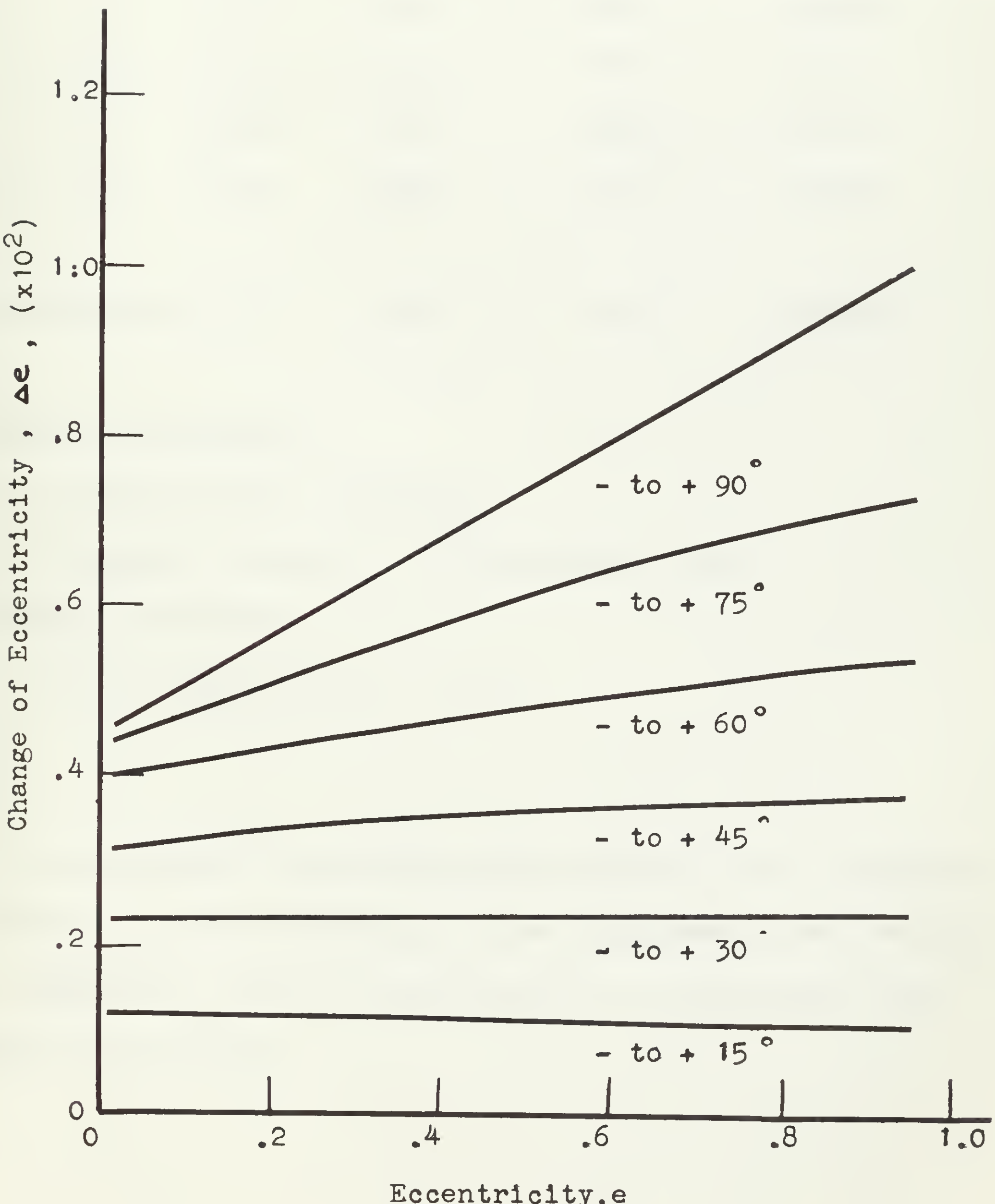


Fig. 3-7

TABLE 3-1

RESULTS OF VARIOUS ACCELERATION TRAJECTORIES

Thrusting arc (degrees)	Number of orbits	Thrusting time (days)	Total time (days)	final perigeal altitude (nautical miles)
- to + 15	785	3.47	275	119.4
- to + 30	391	3.53	137	196.5
- to + 45	259	3.70	88.5	313.0
- to + 60	192	3.88	64.3	463.0
continuous*	39	8.50	8.50	492,000

Initial Conditions:

eccentricity = .0079437

perigeal altitude = 100 nautical miles

Terminal condition:

final eccentricity = 0.92

* This maneuver[1] represents the result of an expanding circular spiral performed by the continuous application of tangential thrust (eccentricity = 0 throughout the entire trajectory).

CHAPTER 4

RENDEZVOUS AND RESUPPLY

4.1 General Considerations

Since the desired acceleration maneuver is to be performed with an unmanned vehicle, it is necessary to consider the rendezvous techniques which would be used to place a crew aboard, resupply expended fuel, supply last minute items for a space voyage, or make last minute adjustments and repairs. It might be argued that the rendezvous problem would as well apply to the circular acceleration maneuver, should the radiation hazards prove sufficient to dictate that the initial phase of acceleration be unmanned.

4.2 Factors Common to Both Acceleration Maneuvers

In order to more fully understand the problems of rendezvous with both the circular and the elliptical trajectories, certain common factors must be postulated at the beginning of the problem. Since the rendezvous may be started from the Earth's surface, the problems of initial launch are common to any rendezvous problem, and are the same problems which have been studied in great

detail by those concerned with the launching of satellites into low Earth orbits.

If launch is to be made from the Earth's surface, directly to the space vehicle, a "parking" orbit need not be established as such. However, first stage burnout may be considered as occurring at a position in space where a low parking orbit could be established. The firing of the second stage could occur immediately, or could be delayed until a later time, utilizing the low parking orbit. It would be impractical to extend the scope of this thesis to include the problems of Earth launch. Hence, the rendezvous problem begins with the ferry vehicle in a low parking orbit, postulated to be circular, with a radius very close to that of the elliptically orbiting space vehicle at perigee.

Clearly, rendezvous can be effected with the space vehicle at any time during the acceleration maneuver. The region of greatest interest, however, is a rendezvous effected with the space vehicle near escape energy. To set the stage for the comparison, an orbital energy of -2.5×10^7 feet^2/sec^2 is selected for the space vehicle with which we desire to rendezvous. This corresponds to a perigeal velocity of 700 feet per second less than that required for escape at a perigeal radius equal to the radius of the Earth plus 200 nautical miles.

4.3 Rendezvous Velocity Requirements

Rider[8], in a recent report, investigated the variation

of transfer orbits from an inner, circular, "parking" orbit to coplanar, outer, elliptic orbits and has shown that, for a general case, the most economical transfer is effected by a rendezvous at the apogee of the final ellipse. However, for the case of the initial radius nearly coincident with the perigee of the final ellipse, the most economical solution is that of rendezvous at perigee of the final ellipse, using one increment of velocity to match the velocity of the space vehicle. Other transfer orbits are possible, but require much greater transfer velocities, since the velocity of the ferry vehicle is not tangent to the velocity of the space vehicle at the point of rendezvous.

For transfer from the inner circular orbit to the spiraling circular type escape trajectory, the Hohmann transfer is the most economical.

Both transfer techniques would require some form of mid-course or terminal guidance. Since a quantitative study of the excess velocity requirements required for guidance is beyond the scope of this thesis, the discussion here is limited to a comparison of the ideal velocity requirements (plus a 20% reserve, in some cases). Qualitatively, the rendezvous at perigee appears to have certain advantages, in that the initial, mid-course, and terminal guidance is lumped into one thrusting phase. Since the two vehicles are in close proximity during the entire transfer phase, small errors should be apparent and should be correctable almost at once. On the other hand,

because of the long transfer time to the circular orbit, small errors which may not be apparent would act for a much longer time, and might require quite a bit more in the way of mid-course or terminal guidance.

Quantitatively considering the ideal velocity requirements, it is apparent that the velocity required for rendezvous with the space vehicle at the perigee of the ellipse is the difference between the circular satellite velocity and the perigeal velocity of the space vehicle. This velocity increment is 9,800 feet per second. For a circling space vehicle to have the specified energy, it must have a circular radius of 2.933×10^8 feet, and the corresponding Hohmann transfer velocity increment is 13,470 feet per second.

Clearly, this represents an ideal velocity requirement of 3,670 feet per second less to rendezvous with the elliptical orbit, a saving which can be realized as payload placed aboard the space vehicle.

For a conventional (chemical or nuclear) rocket, the payload ratio, λ, which is the payload weight expressed as a fraction of the initial rocket weight, can be given as

$$\lambda = e^{\frac{-\Delta V_i}{g I_{sp}}} - \delta \tag{4.1}$$

where δ is the dead weight ratio (the weight of the engine and structure expressed as a fraction of the initial weight).

Assuming dead-weight ratios of 0.1 for a chemical

rocket and 0.3 for a nuclear rocket (representative values), (4.1) may be used to calculate the ideal payloads which may be transferred to either the circling space vehicle or the space vehicle in the elliptical trajectory. This payload capability is shown in Fig. 4-1.

4.4 Resupply and Return to Low Orbit

In the previous section, a one way resupply mission was considered. It is feasible that such a ferry vehicle might be used to resupply the space vehicle and then be carried along on the interplanetary trip, to be used again. More realistic, however, is a two way resupply mission in which the ferry vehicle would rendezvous with the space vehicle, resupply it, and then return to its low orbit for recovery. Since the ferry vehicle is sure to be large and expensive, the desire is likely to be for a recoverable, hence reuseable, vehicle. Since the return to the low Earth orbit would require the same velocity increment as the rendezvous with the space vehicle, the total ideal velocity required for the resupply maneuver is double that required for the one way mission.

In an effort to be realistic, a 20% reserve velocity requirement is added to the ideal velocity, to cover the need for mid-course and terminal guidance. The representative values of specific impulse used in the calculations are 350 seconds for a chemical stage, and 800 seconds for a nuclear stage. The same dead weight ratios (.1 for the chemical and .3 for the nuclear) are used. Using these

values in (4.1), it is apparent the a single stage vehicle is inadequate to provide the required velocity increments to rendezvous with the circling space vehicle. Because of the lower required velocity increments to rendezvous with the space vehicle in the elliptical orbit, a single stage vehicle is feasible for the mission. By extending (4.1) to cover the case of a two stage vehicle, the curves of Fig. 4-2 were calculated to illustrate the capabilities of various ferry vehicles to transfer a payload to the space vehicle and return with a payload to a low Earth orbit.

Fig. 4-2 indicates that it would be possible to resupply the circling space vehicle, but only at the expense of being forced to abandon one stage of the ferry vehicle after rendezvous with the space vehicle. On the other hand, a reuseable, single stage, nuclear powered ferry vehicle would appear to be capable of resupplying the space vehicle in the elliptical trajectory. Such a ferry vehicle is within technological expectations for a time period at which an interplanetary space vehicle might be considered.

4.5 Mission Time for Resupply

To this point, the discussion has been limited to the fuel savings and weight advantages of resupply or transfer to the elliptically orbiting space vehicle as compared to the resupply of the circling space vehicle. It might be argued that one advantage of the circling orbit is the ability to place a repair crew aboard for any given period of time, since rendezvous with and departure from the

circular orbit can be accomplished at any point in the orbit, whereas the period of time spent aboard the elliptically orbiting space vehicle is limited to a minimum of one orbital period (72 hours).

Since the space vehicle is designed for an interplanetary journey of months or years, it must have a simulated environment and time spent aboard the space vehicle is spent in comparative comfort. The ferry vehicle, by contrast, is designed for a short flight to ferry crew and supplies, and the emphasis is likely to be on the payload capacity, rather than comfort. Thus, the main consideration is not the total mission time for resupply, but that portion of the time spent aboard the ferry vehicle in a strange environment. In this respect, the resupply of the space vehicle in an elliptical trajectory has the advantage of a very short transfer time.

For any space mission, it must be assumed that some small errors in launch from the Earth's surface will occur. Discrepancies between the true anomaly of the launch point (as referred to the perigee of the ellipse or the point selected to initiate the Hohmann transfer) and the transfer initiation point itself must be corrected. For the case where the launch point is not in the plane of the orbit, non-planar parameters must be adjusted prior to the initiation of the transfer maneuver. Discrepancies in the time of arrival at the transfer initiation point must also be corrected.

In order that all the foregoing errors may be corrected and any orbital adjustments may be performed, the following launch procedure is proposed:

1. Select transfer initiation position and time (for either Hohmann transfer or perigeal transfer).
2. At time -4.5 to -6.0 hours (depending upon the relative position of the launch point and the transfer initiation point), launch ferry vehicle into low Earth orbit.
3. At time -4.5 hours, begin calibration orbit. (Optional. May be omitted if orbital parameters are accurately known.)
4. At time -3.0 hours, begin orbital adjustment for time and non-planar orbital parameters.
5. At time 0, begin interorbital transfer.

As shown by Fig. 4-3 and 4-4, the launch procedure is essentially the same for rendezvous with either the circling or elliptically orbiting space vehicle. The launch problem, in either case, is to have the ferry vehicle at the preselected transfer initiation point at the proper time.

In step 1, considerable latitude in the selection of the transfer initiation point is available for the transfer to the circling space vehicle, since rendezvous may be accomplished at any point in the orbit. The selection of the transfer initiation point for transfer to the space vehicle in an elliptical orbit is restricted to that point

in time and space which is the perigee of the ellipse. Transfer to the ellipse may take place at a point which is slightly off perigee, but only at the expense of a relatively large velocity increment required to adjust the non-tangential velocity of the ferry vehicle. Since the space vehicle reaches perigee only once every 72 hours, the timing of the launch phase is the most important consideration in the rendezvous with the elliptical orbit. Accordingly, step 4 is included for a timing adjustment.

The time consumed in step 2 will depend upon the relative position of the launch point and the transfer initiation point. If the launch point and the transfer initiation point coincide (an unlikely happening), the time consumed in this step would be zero. If the launch point were slightly past the transfer initiation point, almost one orbit would be required to reach the transfer initiation point for the first time.

Although step 3 is optional, it would serve some useful purposes. It would serve as a calibration orbit to accurately determine the adjustments required. It would also be the best orbit for needed adjustments in the inclination and the line of nodes to make the ferry orbit coplanar with either the elliptical orbit, or the Hohmann transfer ellipse.

In step 4, the main adjustment of the orbital parameters would occur. Since the orbit is essentially circular, initially, the main adjustments would consist of

changes in the non-planar parameters, and the time at which the transfer initiation point is reached. Since the ferry orbit is a low Earth orbit, time cannot be gained by shortening the period of the orbit. However, by applying a small velocity increment in passing the transfer initiation point, the orbit can be enlarged such that the period of the new orbit will coincide with the time remaining until the space vehicle reaches perigee. This velocity increment used to adjust the timing actually costs nothing, since the ferry vehicle would reach the transfer initiation point with slightly more than circular velocity and, hence, would require less of a transfer velocity increment.

During this time adjustment orbit, final adjustment of the line of nodes and the inclination may be performed. Upon completion of this orbit, the ferry vehicle and the space vehicle should arrive at the perigee of the ellipse at the same time and with velocities which are tangent. Thus the conditions for transfer initiation have been met.

The proposed program is designed to correct for the worst possible discrepancies between the launch point and the final orbit. For the rendezvous with the circling space vehicle, it is conceivable that the Hohmann transfer could be initiated from the ground. Chances are more remote that the launch point would coincide with the perigeal transit of the space vehicle, so that perigeal transfer could be accomplished from the ground. The proposed program, then, is a variable one in which the time

would vary between zero and 6 hours, depending upon the exact circumstances.

The time which is invariant in this rendezvous problem is the Hohmann transfer time, which is 14.52 hours to the circling space vehicle of the specified energy. Thus, the total transfer time to the elliptically orbiting space vehicle is a maximum of 6 hours from launch, while the total transfer time to the circling space vehicle varies between 14.52 and 20.52 hours, for the one way trip.

Due to the decreased transfer time involved in the rendezvous with the space vehicle in the elliptical orbit, some increase in payload could probably be realized by eliminating the life support equipment required for a much longer transfer time.

4.6 Summary

The rendezvous with the space vehicle in an elliptical orbit has the following advantages over the rendezvous with the circling space vehicle:

1. Less velocity required to rendezvous and return.
2. Ability to resupply the space vehicle using a reuseable, single stage ferry vehicle, and return to low orbit.
3. Ability to carry relatively large payloads to the space vehicle and return a payload to low orbit.
4. Far less time required aboard the ferry rocket for a resupply mission.
5. Short linear distance traveled in the transfer

maneuver, minimizing mid-course and terminal guidance requirements.

Rendezvous with the circling space vehicle has the advantage of flexibility in the choice of the transfer initiation point, and a certain latitude of the timing of the transfer initiation.

Ideal Payload Ratio Transferred to Space Vehicle

For One Way Trip

$$\lambda = \epsilon^{\frac{-\Delta V_i}{g_0 I_{sp}}} - \delta$$

δ = .1 (chemical)

δ = .3 (nuclear)

v_i = 9,800 ft/sec. (ellipse)

v_i = 13,470 ft/sec. (circle)

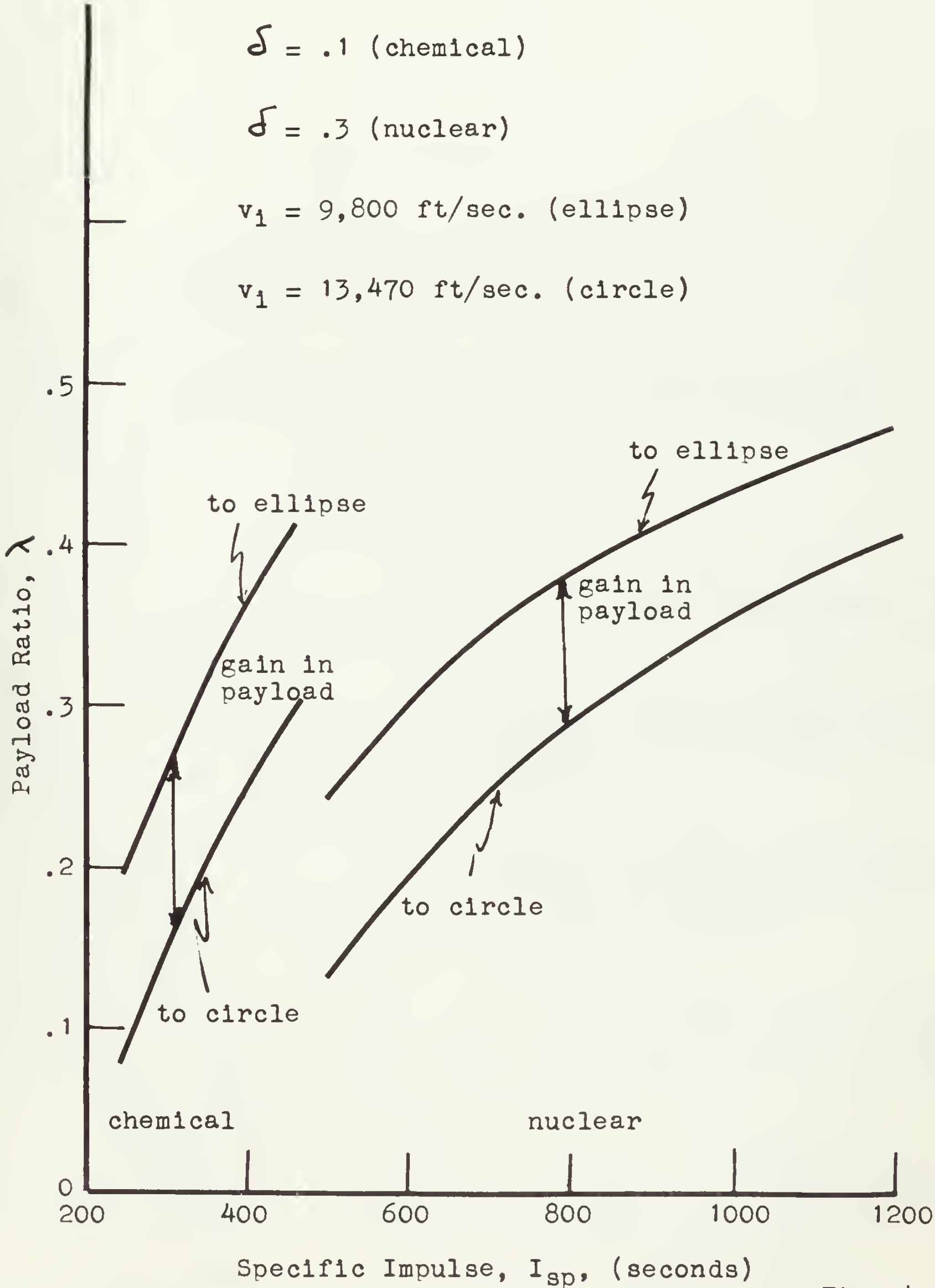


Fig. 4-1

Payload Transferred to Space Vehicle as a

Function of Payload Returned to Earth

Orbital Energy = -24×10^{6} ft^2/sec^2

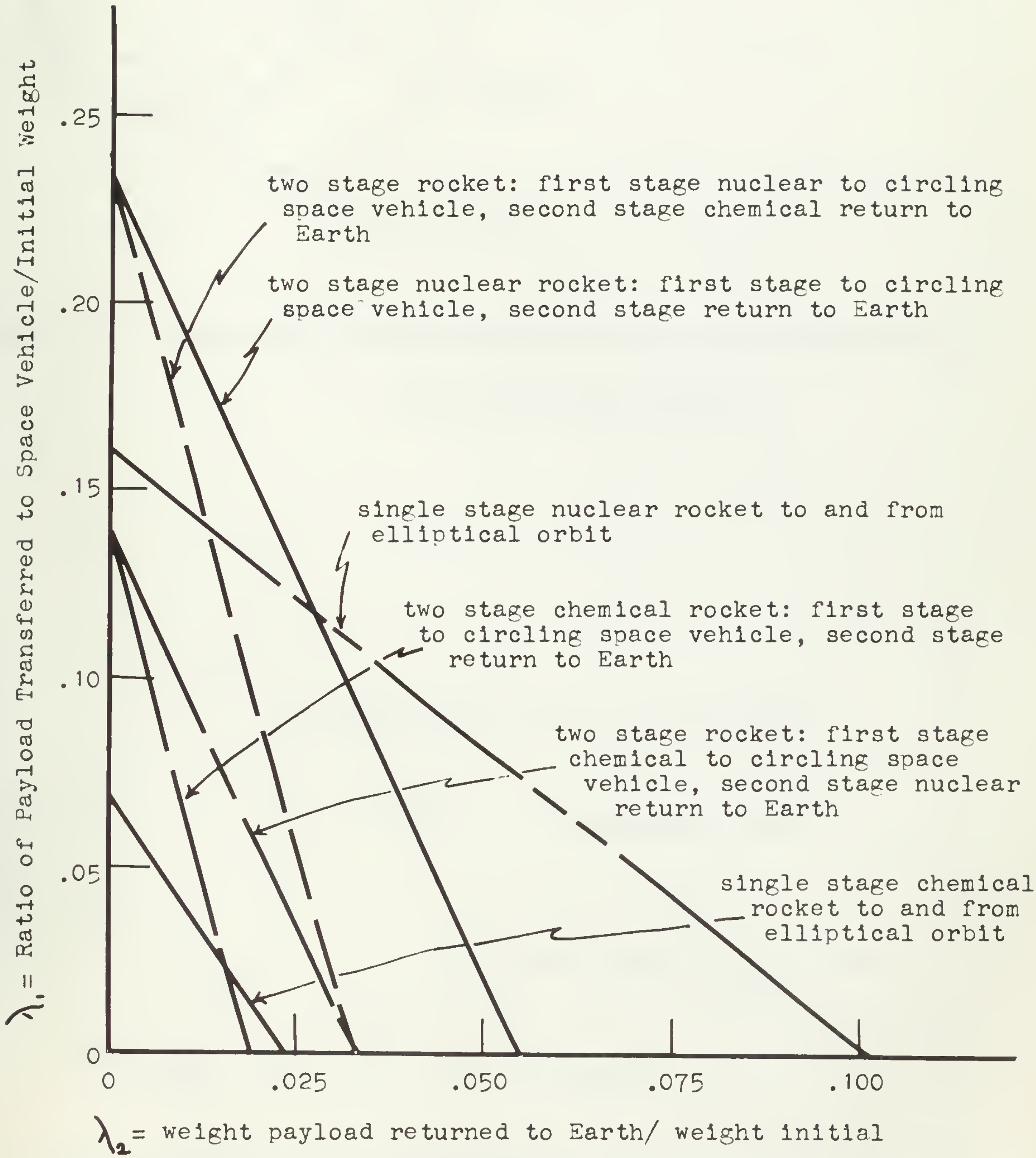


Fig. 4-2

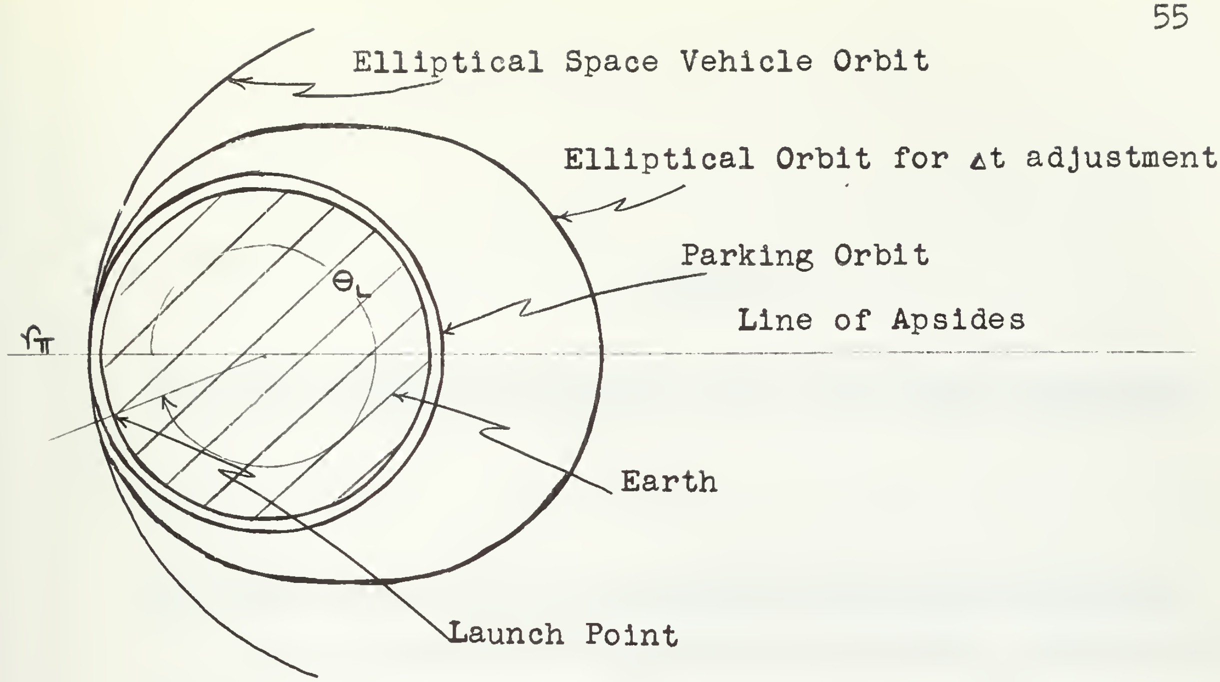


Fig. 4-3 Procedure for Rendezvous Mission with Elliptically Orbiting Space Vehicle

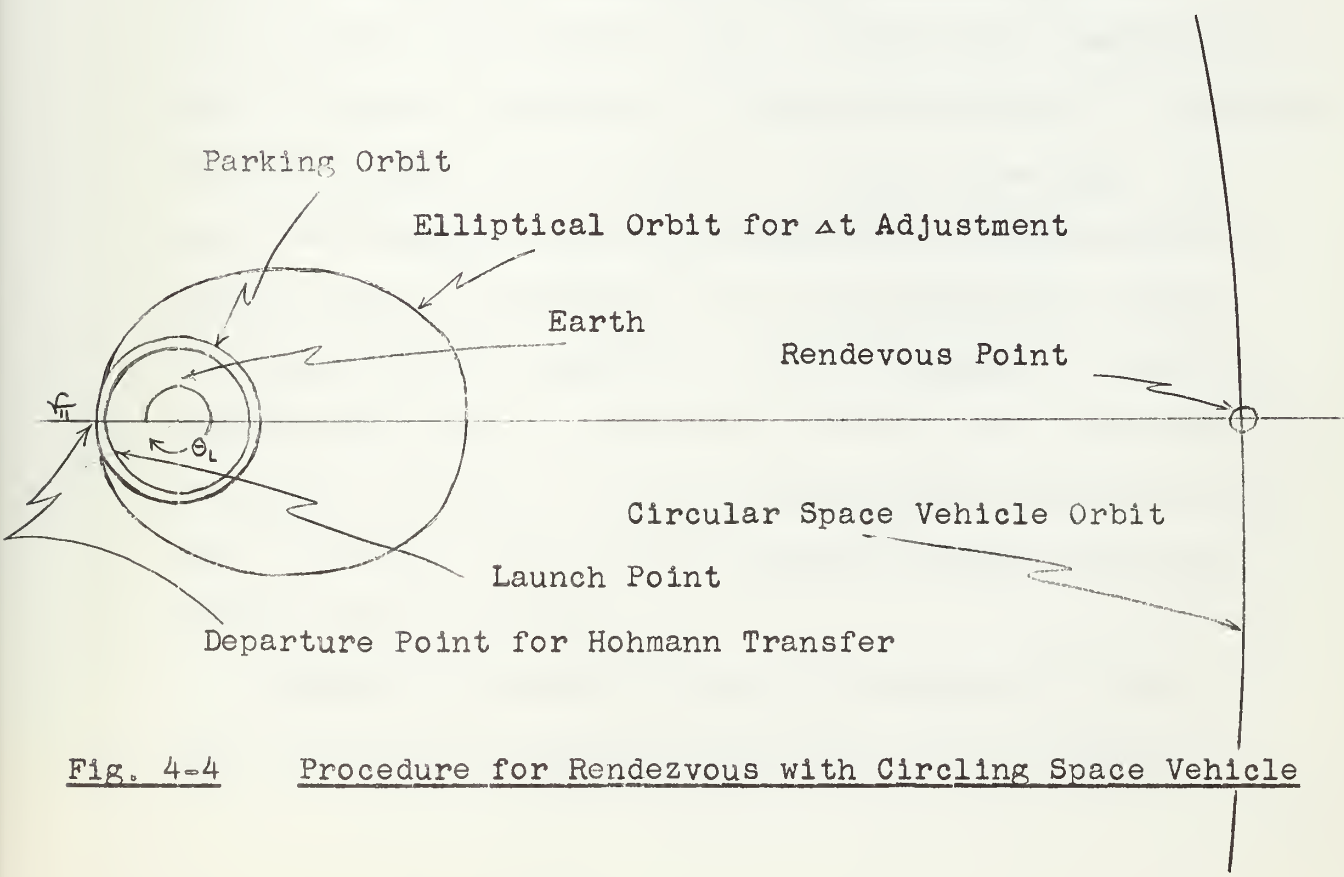


Fig. 4-4 Procedure for Rendezvous with Circling Space Vehicle

CHAPTER 5

FUEL AND WEIGHT ADVANTAGES OF THE ELLIPTICAL TRAJECTORY

5.1 Comparison with the Circular Acceleration Maneuver

In attempting to evaluate the elliptical acceleration maneuver as compared to a conventional low thrust circular escape maneuver, the main advantage of the elliptical maneuver is the increased efficiency of energy addition. This high level of efficiency approaches that of high thrust impulse type chemical energy addition and the reduced amount of expelled mass required for acceleration may be directly converted to increased payload, or decreased initial weight of the vehicle. A second advantage, not quite so apparent, is the possibility of resupplying the vehicle prior to departure on an interplanetary voyage. As demonstrated in Chapter 4, it is feasible to resupply the vehicle prior to departure, not only with crew and perishable supplies, but also with generous quantities of fuel to replace that which is expended during the acceleration maneuver itself.

Langmuir[9] provides a basis for comparison in an expression for the payload capability of a low thrust

space vehicle as a function of the initial acceleration, "a_i",

$$a_i(\alpha) = \frac{.204}{g_0 I_{sp}} \left(e^{-\frac{\Delta V}{g_0 I_{sp}}} - \frac{M_L}{M_i} \right) \tag{5.1}$$

The factor α must be regarded as an effective specific weight, equal to the weight of the powerplant and accelerating system divided by the kinetic energy actually delivered to the expelled mass in one second. Since α is measured in kilogram/kilowatt, it would seem that, for the near future, α will have a value in the range between 1.0 and 100.

Since it is assumed that the comparison between the two different acceleration maneuvers is applicable to the same space vehicle, it is also assumed that the initial acceleration is equal to .001g and the value of α is equal to 1.0.

For the circling escape trajectory, again drawing from Irving's investigations[1], an initial acceleration of .001g requires 8.7 days to escape and corresponds to a total velocity increment of 7.46 km/sec. to reach escape energy.

From the results of Chapter 3, the elliptical trajectory requires a velocity increment between 3.5 and 3.8 km/sec, depending upon the thrust program chosen. A value of 3.75 km/sec is chosen as representative of the required velocity increment for the elliptically accel-

erating space vehicle.

Fig. 5-1 is a comparison, based on (5.1), for the following conditions:

1. A space vehicle using the circular acceleration maneuver with a required escape velocity increment of 7.46 km/sec.
2. A space vehicle using the desired elliptical acceleration maneuver of near constant perigee with an escape velocity increment of 3.75 km/sec.
3. A space vehicle using the elliptical acceleration maneuver, completely resupplied with fuel prior to Earth departure.

The mission velocity requirements of Fig. 5-1 are the total velocity requirements after Earth escape has been accomplished, and the payload ratios are based upon the same initial weight of the vehicle at the start of the acceleration maneuver.

Fig. 5-1 illustrates the payload capability of the different escape maneuvers as a function of the mission velocity requirement, for three different values of specific impulse. To give some definite meaning to the values of Fig. 5-1, the approximate mission velocity requirement of a one year round trip to Mars, including capture and departure at Mars, is noted. The high mission velocity requirements of Fig. 5-1 would include round trips to other planets in this solar system.

It is seen that the advantage of higher efficiency

in the elliptical maneuver, and the advantage of orbital refueling is greatest at low values of total mission velocity requirements and relatively low values of specific impulse. Due to the interchangeability of parameters, this advantage may be realized as an increase in payload for a given vehicle, an increased mission for a given payload, a smaller initial weight for a vehicle to accomplish a given mission, or the use of a powerplant of lower specific impulse to accomplish a given mission. The advantage of the elliptical acceleration maneuver, used in one or more of these categories, coupled with orbital refuelling (which is not too far removed from the state of the art) might make it possible to perform a given mission sooner than would otherwise be possible.

Comparison of Payload Capabilities for Various Mission Requirements for Interplanetary Space Vehicle

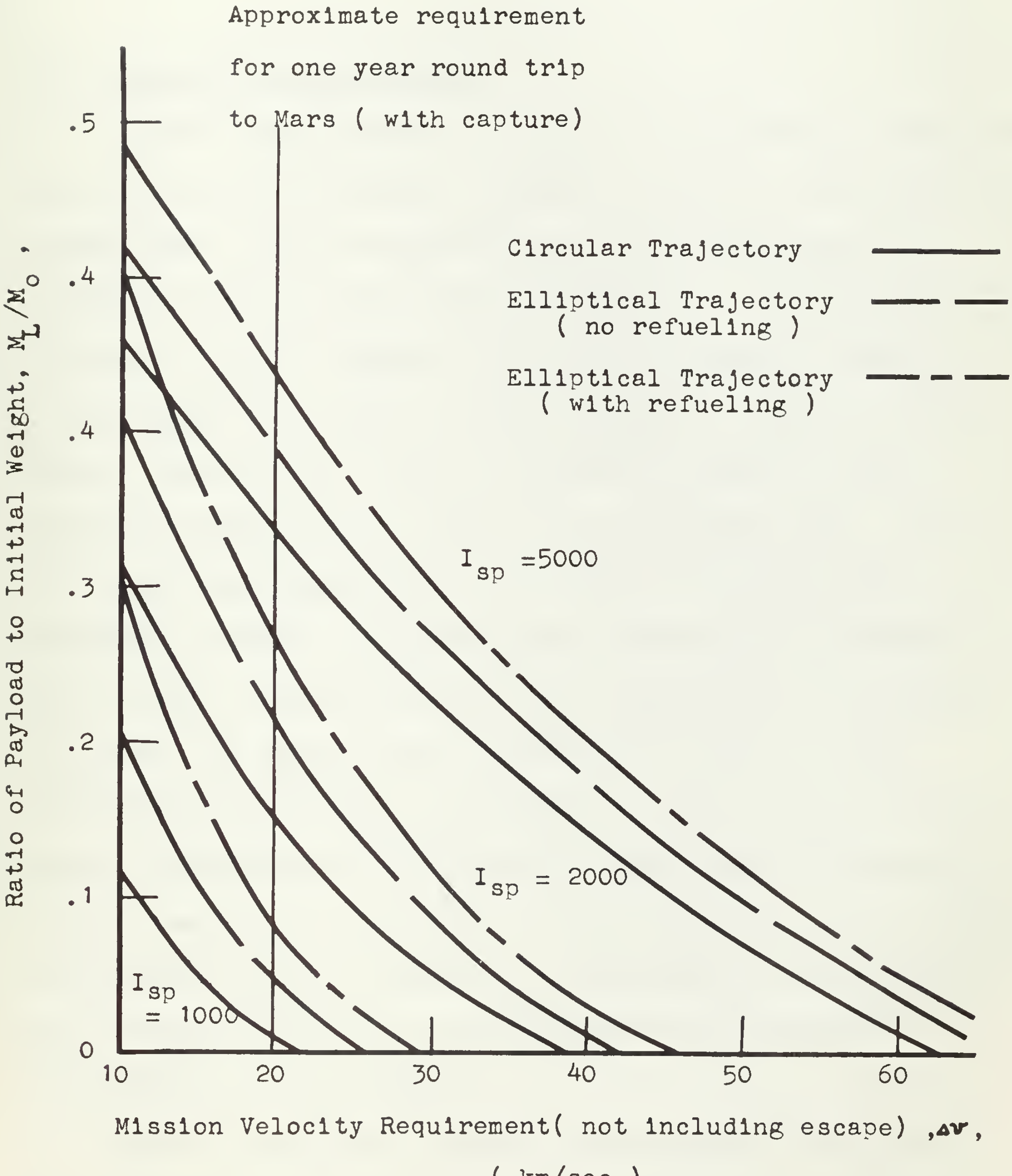


Fig. 5-1

CHAPTER 6

VAN ALLEN RADIATION CONSIDERATIONS

6.1 General Considerations

In the consideration of any low acceleration trajectory, the terrestrial radiation regions, commonly called the Van Allen belts, must be regarded as a possible source of hazard, because of the long acceleration time required for escape. Fig. 3-1 shows a circular trajectory, using an acceleration of .001g, in which escape is achieved in 8.7 days. However, the first 38 orbits are spent within a circular region with a radius of five Earth radii. Fig. 6-1 shows an equatorial planar view of the Van Allen radiation pattern, utilizing data presented by Ehricke[10]. This figure illustrates that the radiation level between 1.2 and 5.75 Earth radii is generally 1000 to 10,000 particles/second/square centimeter. The maximum radiation rate corresponds to 25 Rem/hour. Since much investigation still remains in this area, it would appear that the present discussion must be limited to a qualitative discussion of the relative merits of the circular and elliptical acceleration trajectories.

For an equatorial circular acceleration trajectory, approximately 3.5 days are spent in the Van Allen belts.

Depending upon the exact composition of the radiation, the problem of shielding a manned satellite for the long duration of acceleration in these high radiation regions may prove to be a formidable one[10].

6.2 Advantage of Unmanned Acceleration

The elliptical acceleration trajectory is designed to remain unmanned until escape energy is neared, at which time the space vehicle is in a highly eccentric orbit of low perigee. Since the crew is to be placed aboard after the final orbit has stabilized, and since crew placement is to be achieved at perigee, the result is a very short time spent in the high radiation regions. Superimposed on the Van Allen radiation patterns of Fig. 6-1 is the perigeal portion of the orbit with which rendezvous would be accomplished. For this stable orbit, the space vehicle traverses the high radiation regions in 80 minutes, compared with a total period of 72 hours for the orbit. A crew placed aboard the space vehicle after energy of acceleration has been added, therefore, is exposed to high radiation for a very short time.

The problem of residual radiation picked up by the vehicle during the initial stages of acceleration is one which is essentially common to any type of acceleration maneuver. Some advantage is gained in the elliptical orbit, due to the large amount of time spent at high eccentricities, with large portions of the orbit outside the high radiation regions. During this time, the major

portion of which occurs prior to crew placement, the vehicle would be re-radiating into space any residual radiation picked up during the initial stages of the acceleration.

6.3 Use of a Non-equatorial Acceleration Orbit

Upon consideration of the polar plane view of the Van Allen belts (Fig. 6-2), another possibility suggests itself. Accepting this structure[10,11,12], an expanding elliptical polar trajectory with its axis inclined 15 degrees to the magnetic equator would result in the final ellipse suitable for crew placement, with an orientation illustrated in Fig. 6-2. This trajectory completely avoids all high radiation areas, passing once each orbit through a medium density radiation area (1000 or less particles per second per square centimeter).

By proper programming, escape could be achieved essentially in a plane parallel to the equatorial plane, but displaced vertically three to four Earth radii. Since the space vehicle would accelerate further after escape, the linear displacement from the equatorial plane (which is small compared with interplanetary distances) could easily be corrected by proper mid-course guidance.

The mechanics of the suggested orbit are more devious than this qualitative analysis suggests, but the fact that acceleration thrust is applied for only a small part of each orbit means that a large part of the orbit is available for the adjustment of non-planar orbital parameters. Changes of the orientation of the final orbit,

discussed more fully in Chapter 2, are possible during the acceleration maneuver.

Should the Van Allen radiation areas present a hazard sufficient to make an escape trajectory of the type described desirable, a detailed investigation of this particular trajectory would appear to be a suitable topic for separate study.

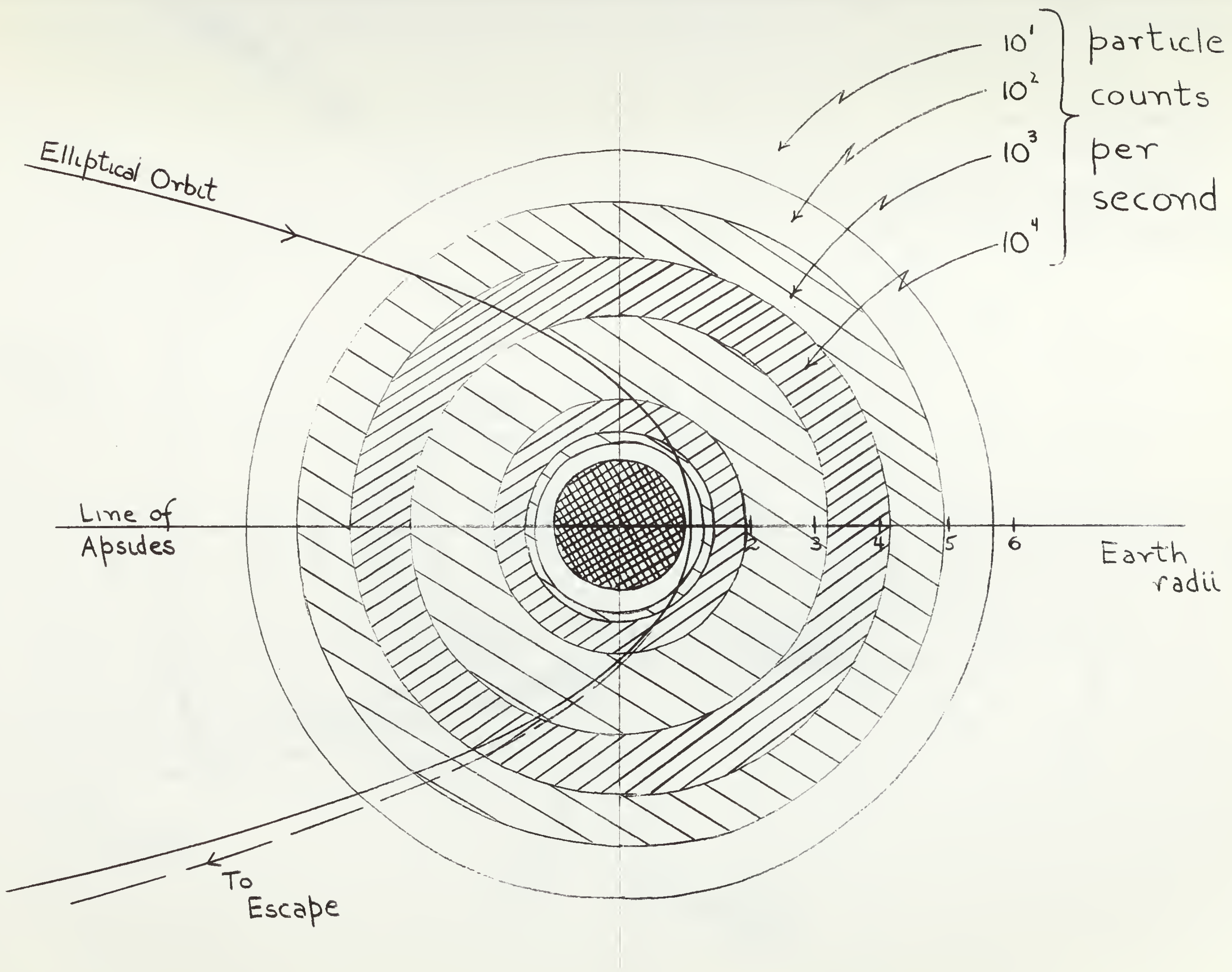


Fig. 6-1 Equatorial Planar View of Circumterrestial Van Allen Radiation Belts

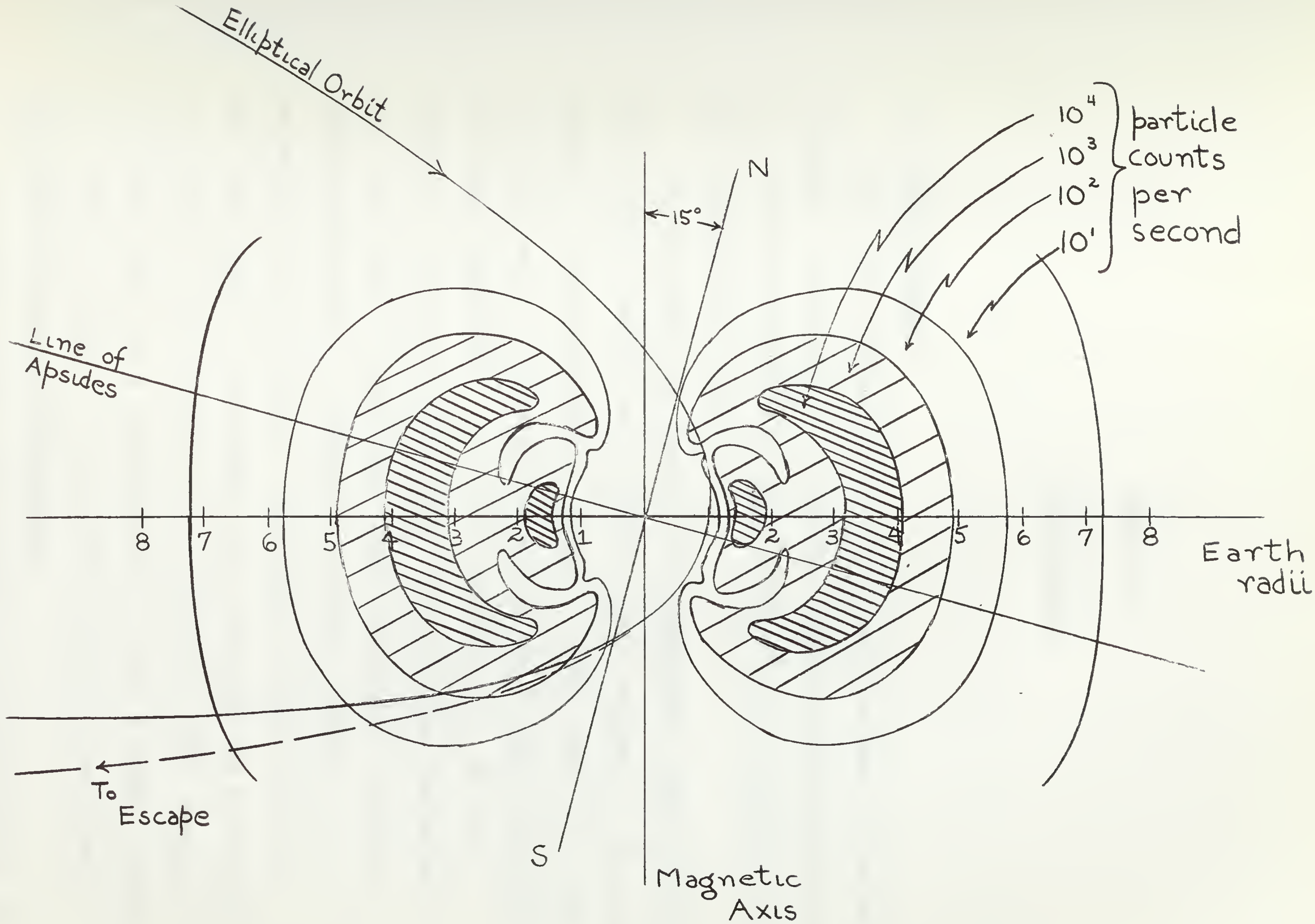


Fig. 6-2 Polar Plane View of Circumterrestial Van Allen Radiation Belts

CHAPTER 7

CONCLUSIONS

7.1 Fuel Economy

The main conclusion of this thesis is that it is both feasible and desirable to use an expanding elliptical orbit of near constant perigee as an acceleration maneuver for a low thrust space vehicle.

By thrusting for small arcs symmetrical about perigee, the perigeal radius can be held nearly constant. The energy added by a given amount of thrust is then added at a high velocity and a low radius, resulting in a high efficiency of energy addition. For thrusting arcs of - to + 60 degrees or less, the elliptical acceleration trajectory uses less than one half the thrust required to add an equivalent amount of energy to a space vehicle in an expanding circular spiral trajectory. Since only one half the thrust is necessary, only one half as much mass is expelled in the acceleration maneuver. This saving in fuel may result in a longer mission time, a larger payload, or a smaller vehicle, or any combination of the three.

7.2 Total Time for Acceleration

In return for the fuel economy of the trajectory, the

total time of the acceleration maneuver is quite long. For an acceleration maneuver in which the perigeal change is held to 10%, the entire maneuver takes 7.5 times as long as a continuous thrust expanding circular spiral using the same acceleration.

7.3 Unmanned Acceleration

One reason for considering an unmanned vehicle during the acceleration maneuver is the total time involved in accelerating to near escape energy. Another is the presence of the circumterrestrial radiation belts. Since much of the acceleration time is spent in the Van Allen belts of high density radiation, the unmanned vehicle would present fewer shielding problems.

7.4 Van Allen Radiation Considerations

The elliptical acceleration maneuver is aptly suited for either an equatorial or polar trajectory. The equatorial trajectory, after acceleration to near escape, would have an orbit with perigee between 1.05 and 1.10 Earth radii, escaping the Van Allen high radiation regions which begin at 1.2 Earth radii. The period of this orbit is 72 hours, 80 minutes of which is spent in the high radiation regions.

If a polar trajectory with its axis inclined 15 degrees to the magnetic equator is utilized, the near escape orbit does not pass through any of the high radiation Van Allen regions. Although the mechanics of the polar trajectory are slightly more devious than those of an

equatorial trajectory, the polar plane trajectory is much more desirable for escaping high radiation areas. Escape from this orbit can be accomplished by impulsive thrusting with chemical engines or by continuous low thrust acceleration. Escape from the polar plane orbit is achieved in a plane essentially parallel with the equatorial plane, but displaced vertically three to four Earth radii. This displacement is small compared with interplanetary distances and should be correctable through continued acceleration and mid-course guidance.

7.5 Rendezvous and Resupply

Since considerations dictate an unmanned acceleration trajectory, rendezvous will have to be made with the space vehicle prior to Earth departure. Rendezvous can be made at the perigee of the ellipse. The transfer velocity increment for rendezvous at perigee is 30% less than that required to rendezvous with a circling space vehicle with the same orbital energy.

Preliminary calculations indicate that a single stage ferry vehicle can be used to place the crew aboard the space vehicle and to resupply the space vehicle from a low Earth orbit. Since rendezvous with and departure from the space vehicle is accomplished at perigee, the transfer distance is very short and the result is a reduction of mid-course and terminal guidance and velocity requirements.

7.6 Orbital Refuelling

Since rendezvous must be made with the space vehicle

to place the crew aboard, it is also possible that larger payloads could be carried to the space vehicle after acceleration. Should some of the larger payload be in the form of fuel to replace the mass expelled during the acceleration maneuver, the payload for an interplanetary mission would be considerably increased. The advantage of orbital refuelling may also be realized as an increase in mission capabilities or a smaller space vehicle.

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